Quantum Theory of Condensed Matter

Prof. John Schliemann
Dr. Andrea Donarini

Mo 10:15, 5.0.21
We 16:15, H33

Sheet 1

---

1. Non-interacting jellium model

Consider a metal of volume V at zero temperature and approximate the periodic ion distribution with the jellium model. By neglecting the electron-electron interaction:

a) Calculate the relation between the density of electrons and the Fermi wavelength.

b) Estimate the value of the Fermi wavelength, and Fermi energy for copper knowing that it is monovalent and with a typical interatomic distance of 2Å.

c) Calculate the energy of the ground state of the metal in the jellium model. Which is the energy per particle?

2. Perturbation theory for interacting jellium

Now let us consider the previous model with the effects introduced by the non homogeneous part of the electron-electron interaction.

a) Prove that the Hamiltonian that represents the electron-electron interaction is written, in second quantization, in the form:

$$V_{\mathrm{el-el}} = \frac{1}{2V} \sum_{\vec{k}_1 \vec{k}_2 \vec{q}} \sum_{\sigma_1 \sigma_2} \frac{e^2}{\epsilon_0 q^2} c^\dagger_{\vec{k}_1+\vec{q}\sigma_1} c^\dagger_{\vec{k}_2-\vec{q}\sigma_2} c_{\vec{k}_2\sigma_2} c_{\vec{k}_1\sigma_1}.$$

Hint: Start by considering the Youkawa potential $V^{k_s} = \frac{e^2}{4\pi\epsilon_0 r} e^{-k_s r}$ and take the limit $k_s \to 0$ in the end of the calculation. In the jellium model we can neglect the term with $\vec{q} = 0$. Why?

b) Calculate the energy per particle of the ground state for the jellium model to first perturbation order in the interaction and express the result in terms of the dimensionless measure r_s. *Hint*: Remember that r_s can be defined by the relation

$$\frac{4\pi}{3}(r_s a_0)^3 = \frac{V}{N}$$

where a_0 is the Bohr radius, V is the volume of the metal and N the total number of electrons.

c) By means of the variational principles show that the first order perturbation of the jellium model predicts the stability of the metals.

Frohes Schaffen!