

# Quantum Theory of Condensed Matter

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Mo 10:15, 5.0.21  
We 16:15, H33

## Sheet 6

### 1. Bloch $T^{3/2}$ law

Let us consider a Heisenberg ferromagnet described by the Hamiltonian

$$H = - \sum_{i,j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

In the spin-wave approximation, the spontaneous magnetization of this system at low temperature is given by the expression:

$$\frac{M_0 - M_S(T)}{M_0} = \frac{1}{NS} \sum_{\vec{q}} \frac{1}{\exp[(\beta \hbar \omega(\vec{q}))] - 1}$$

where  $N$  is the total number of interacting moments of spin  $S$ ,  $M_0 = g\mu_B S \frac{N}{V}$  is the magnetization at saturation and  $\hbar\omega(\vec{q}) = 2S\hbar^2(J_0 - J(\vec{q}))$  is the magnon dispersion relation.  $V$  is the volume of the system,  $\mu_B$  is the Bohr magneton and  $J_0$  is given by the relation  $J_0 = \sum_i J_{ij}$ . Derive the Bloch  $T^{3/2}$  law:

$$\frac{M_0 - M_S(T)}{M_0} \sim T^{3/2}.$$

How does the Bloch law change in  $d$  dimensions?

*Hint:* First transform the summation of  $\vec{q}$  into an integral. Discuss the possibility to apply at low temperatures the small  $q$  limit for the magnon dispersion relation

$$\hbar\omega(\vec{q}) = \frac{D}{2S\hbar^2} q^2$$

and the fact that the integral over  $\vec{q}$  can be extended from the first Brillouin zone to the entire  $\vec{q}$  space.

### 2. Simplified Anderson Impurity Model

Let us consider a quantum dot, described by a single interacting level namely the Hamiltonian

$$H = \sum_{\sigma} \varepsilon_0 c_{\sigma}^{\dagger} c_{\sigma} + U n_{\uparrow} n_{\downarrow},$$

in equilibrium with a thermodynamical bath at temperature  $T$  and chemical potential  $\mu$ .

- a) Write the retarded single particle Green's function for the system and derive the equation of motion for it.
- b) In the previous point you have generated a two particle retarded Green's function: do you recognize it? Now write the equation of motion also for this two particle Green's function. It is useful at this point to transform the system of differential equations from the time to the (Fourier conjugated) frequency domain. The system of equations is now algebraic. Calculate the formal solution of the system. The single particle Green's function of the system still depends on a "parameter": which one?
- c) Starting from the solution calculated at the previous point deduce the form of the spectral function.
- d) Given that the Hamiltonian is invariant for spin rotation demonstrate that the following relation holds:  $\langle n_{\uparrow} \rangle = \langle n_{\downarrow} \rangle$ .

- e) Calculate the form of  $\langle n \rangle \equiv \langle n_\uparrow \rangle = \langle n_\downarrow \rangle$  as a function of the on-site energy  $\varepsilon_0$  making use of the relation that connects the average density to the spectral function:

$$\langle n_\sigma \rangle = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} A_\sigma(\omega; \varepsilon_0, \langle n_{\bar{\sigma}} \rangle) n_F(\omega)$$

where  $n_F(\omega)$  is the Fermi-Dirac distribution function. You can assume for simplicity the chemical potential to be 0 without loss of generality.

- f) Make a plot of the average number of electron as a function of the on site energy  $\varepsilon_0$  and of the spectral function  $A_\sigma(\omega; \varepsilon_0)$  as a function of  $\omega$  for different values of  $\varepsilon_0$ . One can distinguish 3 different intervals for  $\varepsilon_0$  corresponding to three different physical situation. Identify the three intervals and comment on the three regimes.

**Frohes Schaffen!**