Quantum Theory of Condensed Matter

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Mo 10:15, 5.0.21
We 16:15, H33

Sheet 7

1. Phonon Green's functions

The Hamiltonian that describes the quantized oscillations of the ionic cristal lattice reads in second quantization

$$H = \sum_{\vec{q}s} \hbar\omega_s(\vec{q}) \left(b^\dagger_{\vec{q}s} b_{\vec{q}s} + \frac{1}{2} \right)$$

where s labels the optical and acoustic branches. We introduce the retarded Green's function

$$G^r_{\vec{q}s}(t) = -i\theta(t)\langle [b_{\vec{q}r}(t), b^\dagger_{\vec{q}r}] \rangle$$

where $\langle \bullet \rangle = \frac{1}{Z} Tr\{\exp(-\beta H)\bullet\}$ is the thermodynamical average.

a) Justify the choice of zero chemical potential for the thermodynamical average.

b) Calculate explicitly the retarded Green's function by means of equation of motion and directly from the definition.

c) Evaluate the internal energy of the system.

2. Mean field approximation of the simplified AIM

Let's consider the same model of the exercise 1 of Sheet 6 but this time solve it in mean field.

a) Write again the equation of motion for the retarded single particle Green's function and factorize the two-particle Green's function as follows:

$$\langle\langle n_{\bar{\sigma}} c_\sigma, c^\dagger_\sigma \rangle\rangle \approx \langle n_{\bar{\sigma}} \rangle \langle\langle c_\sigma, c^\dagger_\sigma \rangle\rangle$$

where

$$\langle\langle A, B \rangle\rangle \equiv -i\theta(t) \int_{-\infty}^{+\infty} \mathrm{d}t \langle \{A(t), B\} \rangle e^{i\omega t}$$

What is the single particle Green's function in this limit? And the spectral function?

b) Using the same relation between average density and Green's function as the one introduced in exercise 1 of Sheet 6 derive the "self-consistency" equations for the two spin average densities on the dot.(N.B. Disregard for the moment the exact relation between the densities for the different spin species.)

c) Solve, in the limit of zero temperature, the equation derived at the previous point. Make a plot of the different densities as a function of the on-site energy ε_0.

Frohes Schaffen!