University of Regensburg Summer Term 2014

Applications of Group Theory

PD Dr. Andrea Donarini

Lectures 9.2.01, Mondays, 14:15

Exercises H34, Wednesdays, 14:00

Sheet 12

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1. Quaternion algebra

Quaternions are the most natural way to treat double groups. The product of two quaternions is given by:

$$[\![a, \mathbf{A}]\!][\![b, \mathbf{B}]\!] = [\![ab - \mathbf{A}\cdot\mathbf{B}, a\mathbf{B} + b\mathbf{A} + \mathbf{A}\times\mathbf{B}]\!]$$

This exercise is meant to get more familiar with the algebra of these numbers.

1. Prove the associative property of the quaternion product.
2. Prove that the product of two pure quaternions is a pure quaternion only if their corresponding (pseudo-)vectors are orthogonal. Interpret the result in terms of binary rotations.
3. Consider the quaternions $\mathbb{A} = [\![a, \mathbf{A}]\!]$ and $\mathbb{B} = [\![b, \mathbf{B}]\!]$ with the conjugation prescription $\mathbb{A}^* = [\![a, -\mathbf{A}]\!]$. Prove that $(\mathbb{A}\mathbb{B})^* = \mathbb{B}^*\mathbb{A}^*$.
4. Prove that the product of two normalized quaternions is a normalized quaternion.
5. Prove that $\mathbb{A}$ is a pure quaternion if and only if

$$\mathbb{A}^* = -\mathbb{A}.$$

2. Multiplication tables of double groups

Using the quaternion algebra calculate the multiplication tables for the groups $\bar{D}_2$ and $\bar{C}_3$. Verify explicitly the validity of the Opechowski's rules in the construction of the class system for the two aforementioned double groups.

Frohes Schaffen!