University of Regensburg Winter Term 18/19

Quantum theory of condensed matter II

Mesoscopic physics (Quantum transport)

Prof. Milena Grifoni	Tue	8:00 - 10:00	9.2.01
	Fri	12:00 - 14:00	9.2.01
PD Dr. Andrea Donarini	Tue	10:00 - 12:00	5.0.21

Sheet 3

1. Electronic waveguide

Consider a narrow conductor etched out of a wide one, as shown in the figure 1. The wide conductor can be treated simply as a two-dimensional conductor.

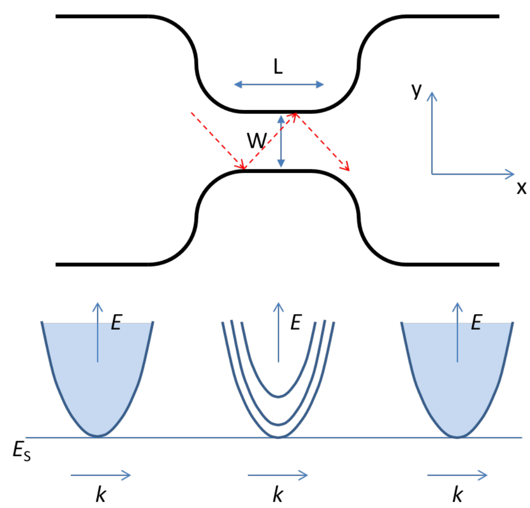


Figure 1: Schematic representation of the constriction in a 2DEG considered in this exercise.

1. Calculate the location of the Fermi energy relative to the bottom of the band E_S, assuming an effective mass $m^* = 0.07m$ with m the electron mass and an electron density of 5×10^{11} cm^{-2}.

(1 Point)

2. Plot the electronic density of states for the central region, assuming $W = 0.1\mu$m. Consider two possible realizations of the confining potential:

   a) Hard walls.

$$U(y) = \begin{cases} 0, & \text{for} \quad -W/2 < y > W/2 \\ \infty, & \text{otherwise} \end{cases}$$

   b) Harmonic confinement.

$$U(y) = \frac{1}{2} m \omega_0^2 y^2$$

   with ω_0 chosen such that $U(y = \pm W/2) = E_F - E_S$.

(2 Points)

3. A classical particle travelling in the etched conductor region will, in general, bounce a few times up and down before reaching again the wide conductor, as illustrated by the red-dashed arrows in figure 1. Is it possible to construct the quantum analogue for this dynamics? How?

(1 Point)

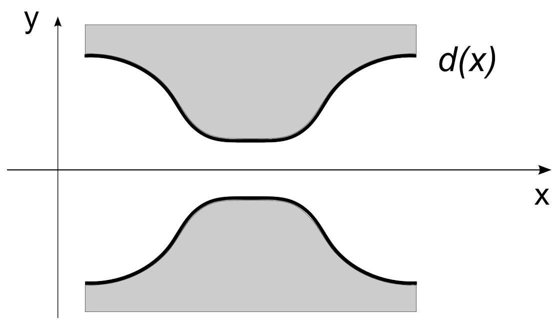
2. Adiabatic quantum point contact

A quantum point contact as shown in the picture below can be described by the two-dimensional Schrödinger equation

$$-\frac{\hbar^2}{2m}\nabla^2\psi(x,y) = E\psi(x,y) \tag{1}$$

with the boundary condition

$$\psi(x, \pm d(x)) = 0 \tag{2}$$



Make the following Ansatz for the wavefunction of the system:

$$\psi(x,y) = \sum_{n=1}^{\infty} c_n(x)\,\phi_n(y;x)$$

where

$$\phi_n(y;x) = \sqrt{\frac{1}{d(x)}}\sin\left(\frac{n\pi}{2d(x)}(y + d(x))\right),$$

are a set of local, basis wave functions for the transverse direction which obviously fulfill the boundary condition (2).

1. Derive a set of equations for the functions $c_n(x)$, by inserting the Ansatz for $\psi(x,y)$ in the Schrödinger equation (1), and by projecting it on the basis state $\phi_n(y;x)$.

2. Under which conditions for the function $d(x)$ are the equations for $c_n(x)$ and $c_m(x)$ with $m \neq n$ independent? Give a physical interpretation of the result.

Frohes Schaffen!