Quantum theory of condensed matter II

Mesoscopic physics (Quantum transport)

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Tue	8:00 - 10:00	9.2.01
Fri	12:00 - 14:00	9.2.01
Tue	10:00 - 12:00	5.0.21

Sheet 4

1. Landau levels

Consider a 2 dimensional conductor confined to the xy plane in presence of a uniform magnetic field $\vec{B} = B\vec{e}_z$ pointing in the z direction.

1. Prove that the Schrödinger equation for the system can be written in the form:

$$\left(\frac{(p_x + eBy)^2}{2m} + \frac{p_y^2}{2m}\right)\Psi(x,y) = E\Psi(x,y) \tag{1}$$

   where $p_x = -i\hbar\frac{\partial}{\partial x}$ and $p_y = -i\hbar\frac{\partial}{\partial y}$.
   Hint: Use the minimal coupling $\vec{p} \to \vec{p} - e\vec{A}$ for the description of the magnetic field, where $\vec{A}$ is the vector potential. **2 Points**

2. Using the translational invariance of the Hamiltonian (1) in the x direction one can make the Ansatz

$$\Psi(x,y) = \frac{1}{\sqrt{L}} e^{ikx}\chi_k(y),$$

   where L is the (large, see next point for a quantitative estimate) size of the conductor, both in the x and y direction. Prove that the function $\chi_k(y)$ should solve the equation of a quantum harmonic oscillator of mass m and frequency $\omega_c = \frac{|e|B}{m}$ centered around the point $y_k = -\frac{\hbar k}{|e|B}$. Write the corresponding eigenvalues $E_{n,k}$ and also the eigenfunctions $\chi_{n,k}(y)$ in terms of the Hermite polynomials. **4 Points**

3. Prove that the solutions obtained above is only valid in the limit $L \gg \sqrt{\hbar/(m\omega_c)}$ and $L \gg |\hbar k/(eB)|$, thus giving a quantitative meaning to the condition on L assumed in the previous point. **2 Points**

4. Which is the group velocity associated to the state $\chi_{n,k}(y)$? How does this result compare with the classical picture of the orbits of a charged particle in a magnetic field?

5. Now consider the system considered before but with an additional parabolic confinement $U(y) = \frac{1}{2}m\omega_0^2 y^2$. How are the solutions of the Schrdinger equation modified by the additional confinement? Which is the group velocity associated to the state of the system?

2. Filling of the Landau levels

Consider a narrow conductor etched out of a wide conductor by a parabolic confining potential. Calculate the number of transverse modes as a function of the magnetic field, for the following cases:

a) assuming constant Fermi energy

b) assuming constant electron density

Use the results for the spectrum of the Landau levels obtained in the first exercise and prove that if the Fermi energy remains constant then the conductor can be completely depleted as the magnetic field is increased, (see B.J. van Wees *et al.*, *Phys. Rev. B* **38**, 3625 (1988) for the experimental evidence), but if the electron density is assumed to remain constant then the number of modes cannot decrease to zero (at least one mode remains always occupied) as it is shown, for example in K.F. Berggren *et al.*, *Phys. Rev. B* **37**, 10118 (1988).

Frohes Schaffen!