University of Regensburg | Winter Term 18/19

Quantum theory of condensed matter II

Mesoscopic physics (Quantum transport)

Prof. Milena Grifoni	Tue	8:00 - 10:00	9.2.01
	Fri	12:00 - 14:00	9.2.01
PD Dr. Andrea Donarini	Tue	10:00 - 12:00	5.0.21

Sheet 14

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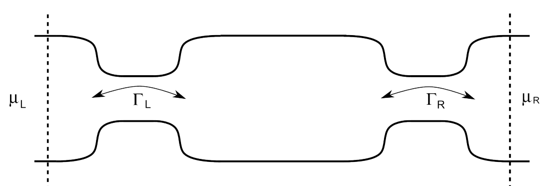
1. General questions

i) Dynamical regimes. Mesoscopic and nanoscopic systems are characterized by different length scales which in turn determine the kind of transport regimes one can encounter.

a) Under which conditions is a mesoscopic conductor ballistic and when is it diffusive?

b) Consider a ballistic double barrier structure



and the energy scales provided by the temperature $k_\mathrm{B}T$, the tunnelling rates Γ_L, Γ_R at the two barriers, and the charging energy U. Which dynamical regimes are observed depending on the relative magnitude of these parameters ?

ii) Provide two explicit examples for the physical realization of:

a) A quantum point contact

b) A quantum dot.

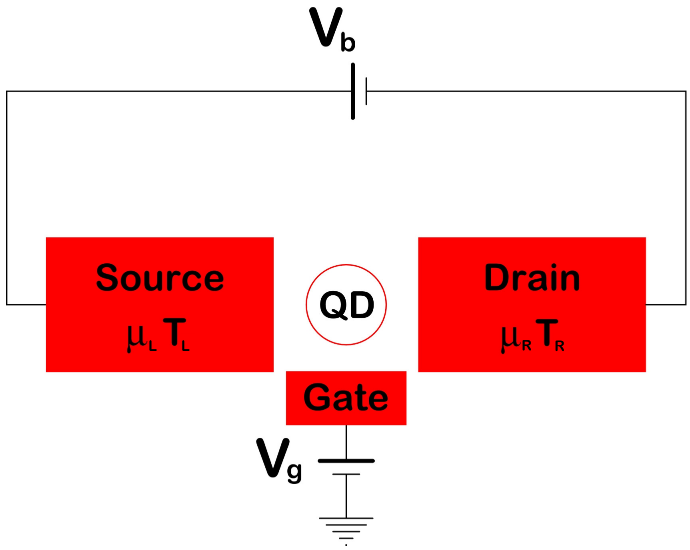
Sketch the associated conductance G as a function of the (plunger) gate voltage V_g. Comment on possible effects of temperature.

2. Single electron transistor

The figure below is a schematic representation of a single electron transistor: A quantum dot is coupled to a source, a drain and a gate electrode. We describe the system via the Anderson impurity model tunnel coupled to two leads as discussed in the lecture. The chemical potentials difference of the leads can be tuned by an applied bias potential, i.e. $eV_\mathrm{b} = \mu_\mathrm{L} - \mu_\mathrm{R}$, where $e > 0$ is the modulus of the electron charge and $\mu_\mathrm{L}(\mu_\mathrm{R})$ is the left(right)-lead chemical potential. The potential drop across the structure depends on the capacitive coupling between the dot and the leads. Moreover, we introduce the effect of the gate via a modification of the Anderson Hamiltonian

$$\hat{H}_\mathrm{S} = \sum_\sigma \left(\varepsilon_\mathrm{d} - e\alpha_\mathrm{G}V_\mathrm{G}\right) \hat{d}_\sigma^\dagger \hat{d}_\sigma + U\hat{N}_\uparrow \hat{N}_\downarrow,$$

where V_G is the electrostatic potential of the gate electrode, α_G is the gate coupling constant and $\hat{N}_\sigma$ is the dot number operator per spin species. Assume moreover that the bias voltage drops symmetrically with respect to the dot, i.e. $\mu_\mathrm{L} = \mu_0 + eV_\mathrm{b}/2$ and $\mu_\mathrm{R} = \mu_0 - eV_\mathrm{b}/2$.



1. Following the procedure we adopted in the previous Sheet, justify that the master equation for the reduced density matrix assumes the following form:

$$\dot{P}_0 = - \sum_{\alpha=L,R} \gamma_\alpha \left[2 f_\alpha^+(\varepsilon_\mathrm{d}) P_0 - \sum_\sigma f_\alpha^-(\varepsilon_\mathrm{d}) P_{1\sigma} \right],$$

$$\dot{P}_{1\sigma} = - \sum_{\alpha=L,R} \gamma_\alpha \left[\left(f_\alpha^+(\varepsilon_\mathrm{d}+U) + f_\alpha^-(\varepsilon_\mathrm{d}) \right) P_{1\sigma} \right] + \\ + \sum_{\alpha=L,R} \gamma_\alpha \left[f_\alpha^+(\varepsilon_\mathrm{d}) P_0 + f_\alpha^-(\varepsilon_\mathrm{d}+U) P_2 \right],$$

$$\dot{P}_2 = - \sum_{\alpha=L,R} \gamma_\alpha \left[2 f_\alpha^-(\varepsilon_\mathrm{d}+U) P_2 - \sum_\sigma f_\alpha^+(\varepsilon_\mathrm{d}+U) P_{1\sigma} \right],$$

where $f_\alpha^+(\varepsilon) \equiv [1 + e^{\beta_\alpha(\epsilon - \mu_\alpha)}]^{-1}$ and $f_\alpha^-(\varepsilon) = 1 - f_\alpha^+(\varepsilon)$. **(2 Points)**

2. Verify that the conditions for allowed tunnelling derived in the lecture for the sequential tunnelling regime are the same as the ones in which the Fermi functions show an inflection point. Give a physical interpretation of the result. **(2 Points)**

3. Prove that the current flowing from the α-th bath towards the impurity is given by

$$I_\alpha = \gamma_\alpha \sum_\sigma \left\{ f_\alpha^+(\varepsilon_\mathrm{d}) P_0 + \left[f_\alpha^+(\varepsilon_\mathrm{d}+U) - f_\alpha^-(\varepsilon_\mathrm{d}) \right] P_{1\sigma} - f_\alpha^-(\varepsilon_\mathrm{d}+U) P_2 \right\}$$

Hint: Consider the definition of the current as the average particle variation on the impurity. **(2 Points)**

4. Sweeping the gate voltage one can change the electron number one by one. Determine the gate values at which the number of electrons in the dot changes. In such "resonant" conditions calculate the conductance. Compare to the conductance calculated with the equation of motion method for the Green's functions (Sheet 11).

Hint: Define the current as $I = (I_L - I_R)/2$ and use the current conservation condition ($I_L = -I_R$).

Frohes Schaffen!