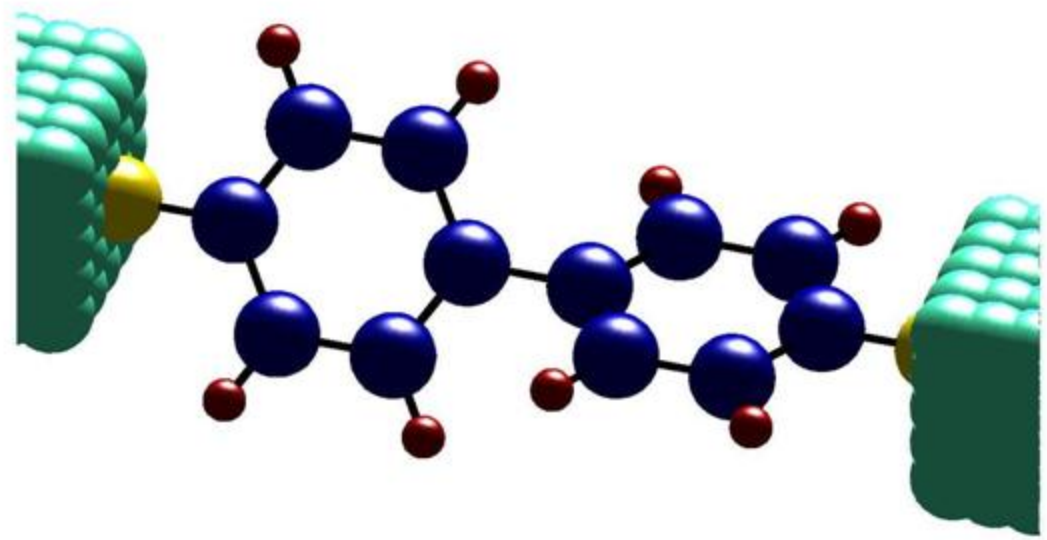




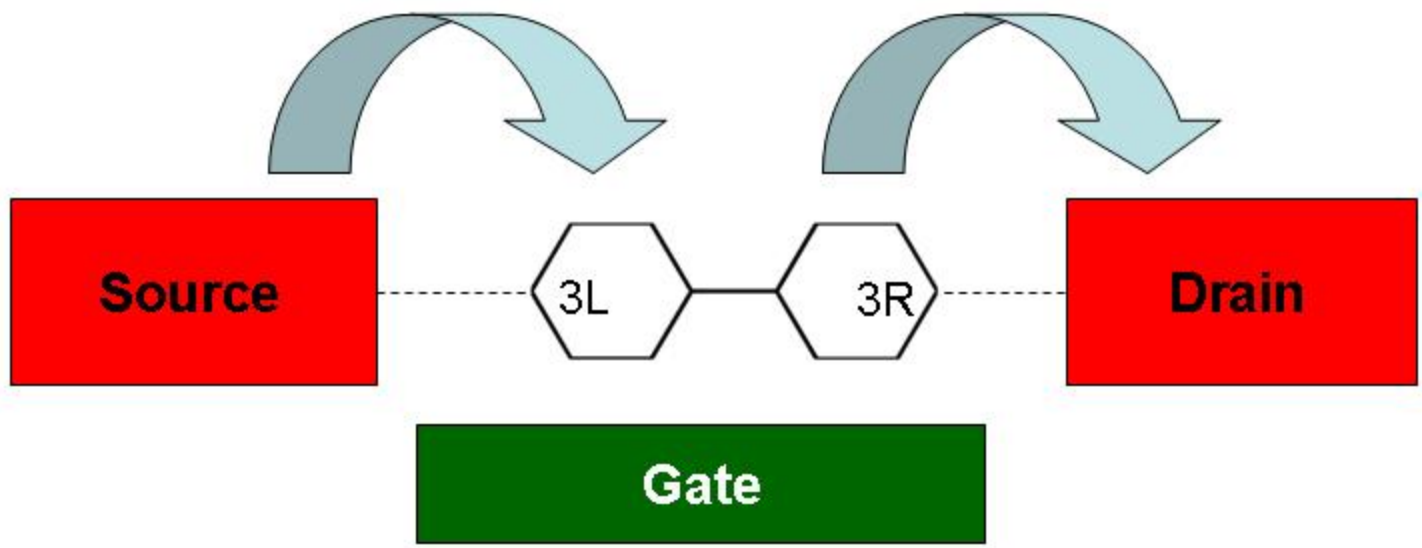
# Electromechanical properties of a biphenyl transistor

*Andrea Donarini*





# The model



- Weak coupling to the leads + low temperature -> Coulomb blockade
- Gate voltage
- Torsional (the softest mechanical ) degree of freedom of the molecule





# The Hamiltonian

The hamiltonian of the device can be written as

$$H = H_{Mol} + H_{Leads} + V$$

Where

$$H_{Mol} = T_{\theta} + H_{PPP}(\hat{\theta})$$

$$H_{Leads} = \sum_{\alpha k \sigma} \epsilon_{k, \alpha} c_{\alpha k \sigma}^{\dagger} c_{\alpha k \sigma}$$

$$V = t \sum_{\alpha k \sigma} (c_{\alpha k \sigma}^{\dagger} c_{\alpha 3 \sigma} + c_{\alpha 3 \sigma}^{\dagger} c_{\alpha k \sigma})$$



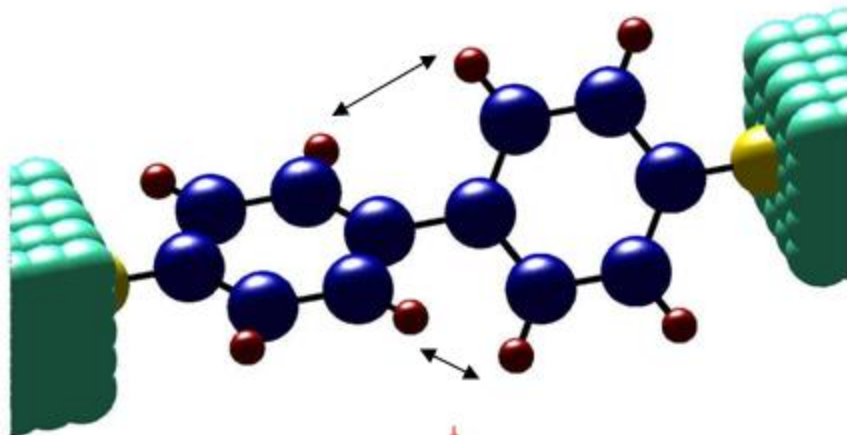


# Isolated biphenyl

- $H_{PPP}$  is the Pariser-Parr-Pople Hamiltonian

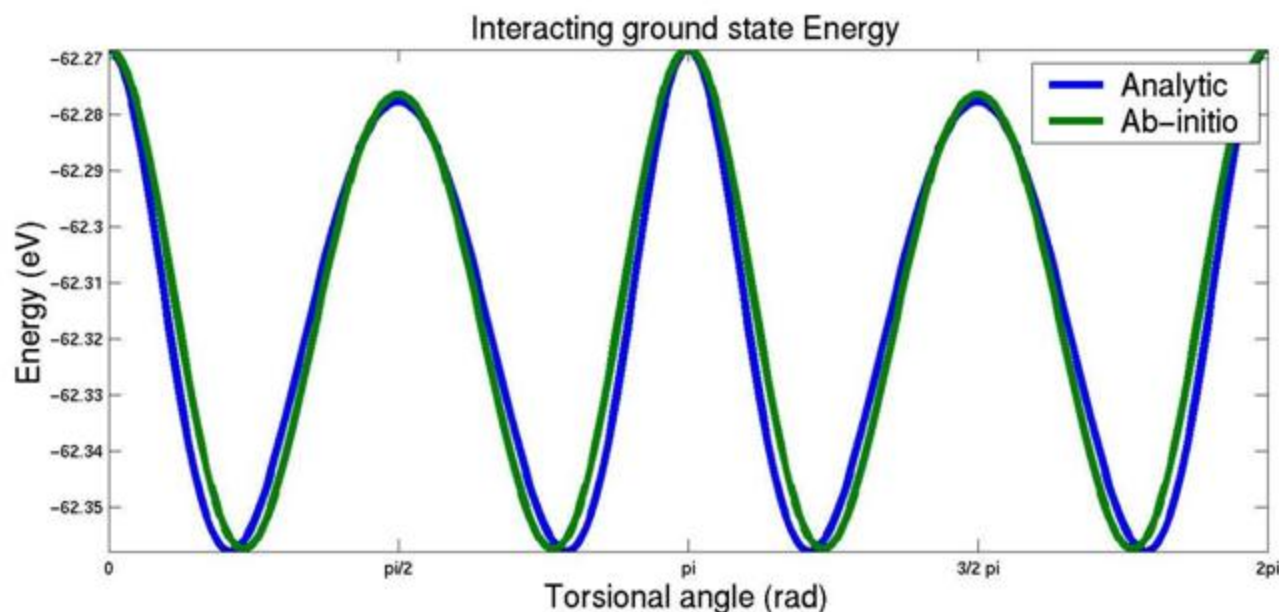
$$H_{PPP}(\theta) = \sum_{i\sigma} b_{ii+1}(\theta)(c_{i\sigma}^\dagger c_{i+1\sigma} + c_{i+1\sigma}^\dagger c_{i\sigma}) + \sum_i U_i \left( \hat{n}_{i\uparrow} - \frac{z_i}{2} \right) \left( \hat{n}_{i\downarrow} - \frac{z_i}{2} \right) \\ + \sum_{\langle i < j \rangle} V_{ij}(\theta)(\hat{n}_i - z_i)(\hat{n}_j - z_j)$$

+ H-H steric repulsion (Lehnard-Jones potential)





# Neutral state



- ground state electronic energy for the neutral molecule:  
four **tilted equilibrium configurations**
- Technique: Hartree-Fock approximation of the PPP Hamiltonian



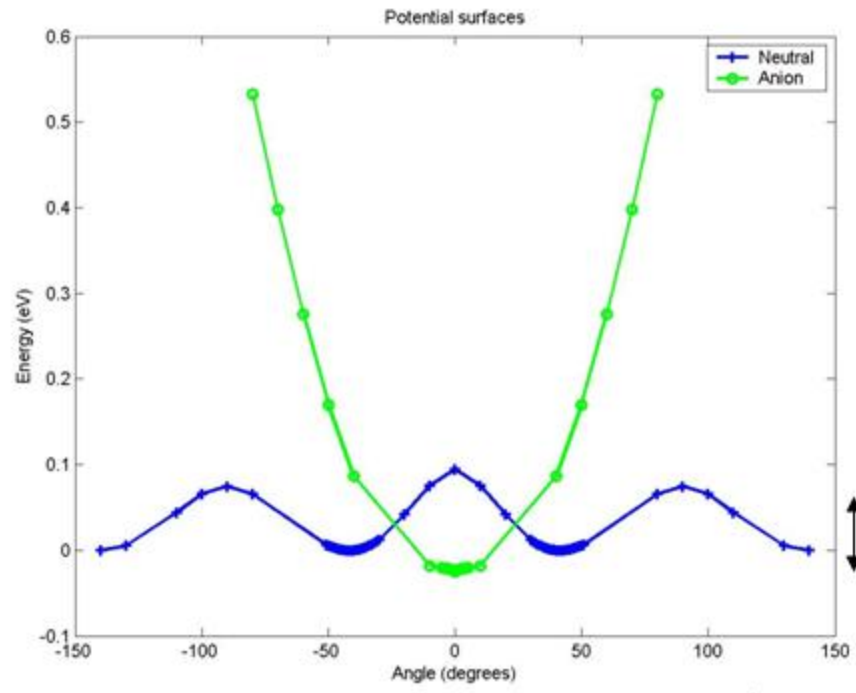


# Anionic state

- Planar equilibrium configuration for the anionic state
- The Hartree-Fock approximation fails for the anionic state

**CORRELATIONS!!!**

(work in progress to go beyond Hartree-Fock )



DFT calculations  
With gaussian basis set  
And B3LYP exchange potential

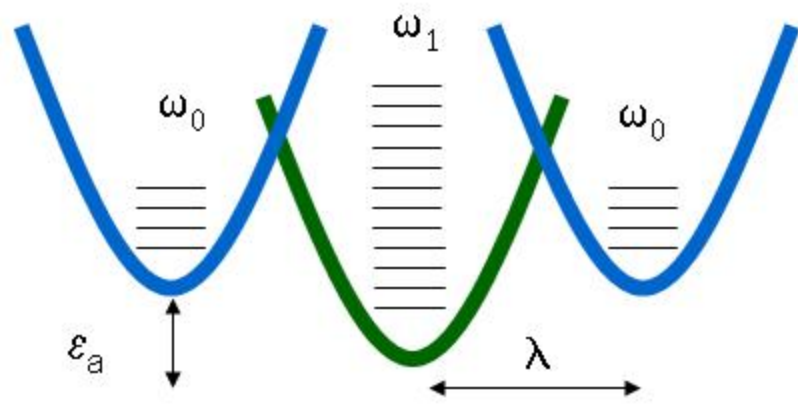


0.1 eV





# Model ... of the model



- Harmonic approximation for the anionic state
- Low laying states of a double (harmonic) well potential for the neutral state
- The model is identified by three parameters:

$$\alpha = \sqrt{\frac{\omega_1}{\omega_0}} \quad \lambda = \frac{\Delta\theta}{\sqrt{\theta_{z0}\theta_{z1}}} \quad \epsilon_a = \frac{EA}{\hbar\sqrt{\omega_0\omega_1}}$$





# Definition of Unity

- We write the previous approximation in terms of unity operator:

$$\mathbf{1} = |N\rangle\langle N|(\mathcal{P}_+ + \mathcal{P}_-) + |N+1\rangle\langle N+1|\mathcal{P}_0$$

- where  $\mathcal{P}_\pm = \sum_{n=0}^{N_\pm} |n_\pm\rangle\langle n_\pm|$  ,  $\mathcal{P}_0 = \sum_{n=0}^{\infty} |n_0\rangle\langle n_0|$

that define the **effective Hilbert space**.







# Effective Hamiltonian

$$\begin{aligned}
H_{eff} = & \sum_{\alpha k \sigma} \epsilon_{k,\alpha} c_{\alpha k \sigma}^\dagger c_{\alpha k \sigma} + \\
& + \sum_{\alpha k \sigma, n, m} \left[ t_{n,m}^{(+)} c_{\alpha k \sigma}^\dagger c_{d \sigma} |n_+\rangle \langle m_0| + t_{n,m}^{(-)} c_{\alpha k \sigma}^\dagger c_{d \sigma} |n_-\rangle \langle m_0| + t_{n,m}^{(+)} c_{d \sigma}^\dagger c_{\alpha k \sigma} |m_0\rangle \langle n_+| + t_{n,m}^{(-)} c_{d \sigma}^\dagger c_{\alpha k \sigma} |m_0\rangle \langle n_-| \right] + \\
& + \left( 1 - \sum_{\sigma} c_{d \sigma}^\dagger c_{d \sigma} \right) \left\{ \epsilon_0 + \hbar \omega_0 \left[ \mathcal{P}_+ \left( d_+^\dagger d_+ + \frac{1}{2} \right) \mathcal{P}_+ + \mathcal{P}_- \left( d_-^\dagger d_- + \frac{1}{2} \right) \mathcal{P}_- \right] \right\} + \\
& + \sum_{\sigma} c_{d \sigma}^\dagger c_{d \sigma} \left[ \epsilon_1 + \mathcal{P}_0 \left( d^\dagger d + \frac{1}{2} \right) \mathcal{P}_0 \right]
\end{aligned}$$

where

$$c_{d \sigma}^\dagger |N\rangle \equiv |N + 1, \sigma\rangle$$

$$t_{n,m}^{(\pm)} = t \underbrace{\langle N | c_{3\alpha} | N + 1 \rangle}_{\substack{\text{We assume it independent of } \theta}} \langle n_{\pm} | m_0 \rangle$$

Franck-Condon  
coefficient





# Generalized Master Equation

We study the dynamics of the system with the Generalized Master Equation:

$$\dot{\sigma} = -\frac{i}{\hbar}[H_{Mol}, \sigma] - \frac{1}{\hbar^2} \int_0^\infty d\tau \text{Tr}_{Leads} \{ [V, [\tilde{V}_I(-\tau), \sigma]] \otimes \rho_{Leads} \}$$

- coherences between **different charge states** vanish
- due to the mechanical degeneracies of the neutral state we **MUST** keep **coherences between displaced mechanical states**
- we write the GME in the basis that diagonalize the molecule hamiltonian (Bloch-Redfield form)

$$\dot{\sigma}_{ij} = \sum_{mn} R_{ijmn} \sigma_{mn}$$





# GME (II)

The Redfield tensor  $R_{ijmn}$  depends on

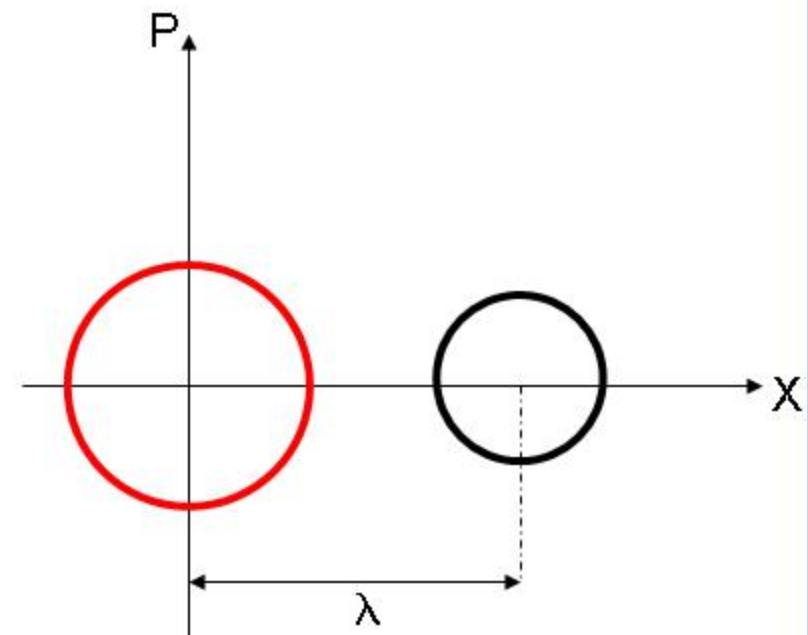
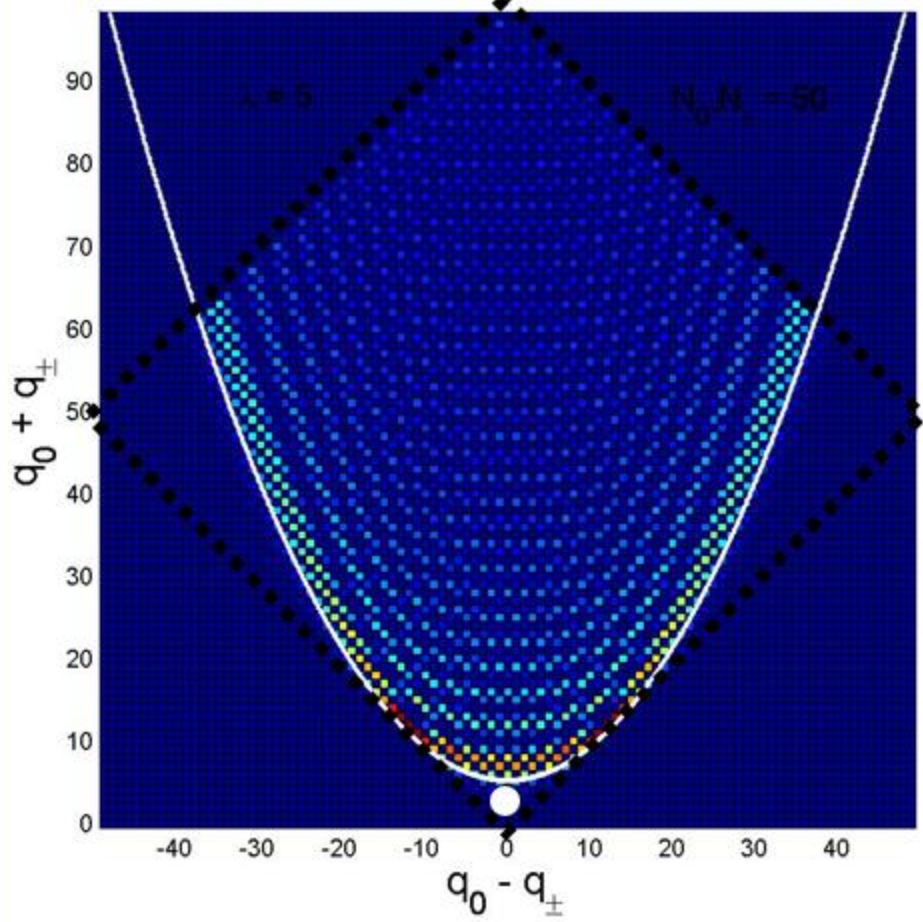
- **Fermi factors** - the Pauli exclusion principle prevents some transitions to occur
- **Bare tunneling rates** - given by the density of states in the leads times the electrical coupling leads-molecule
- **Franck-Condon coefficients** - each state transition in the system is electromechanical





# The Franck-Condon parabola

Franck Condon rates



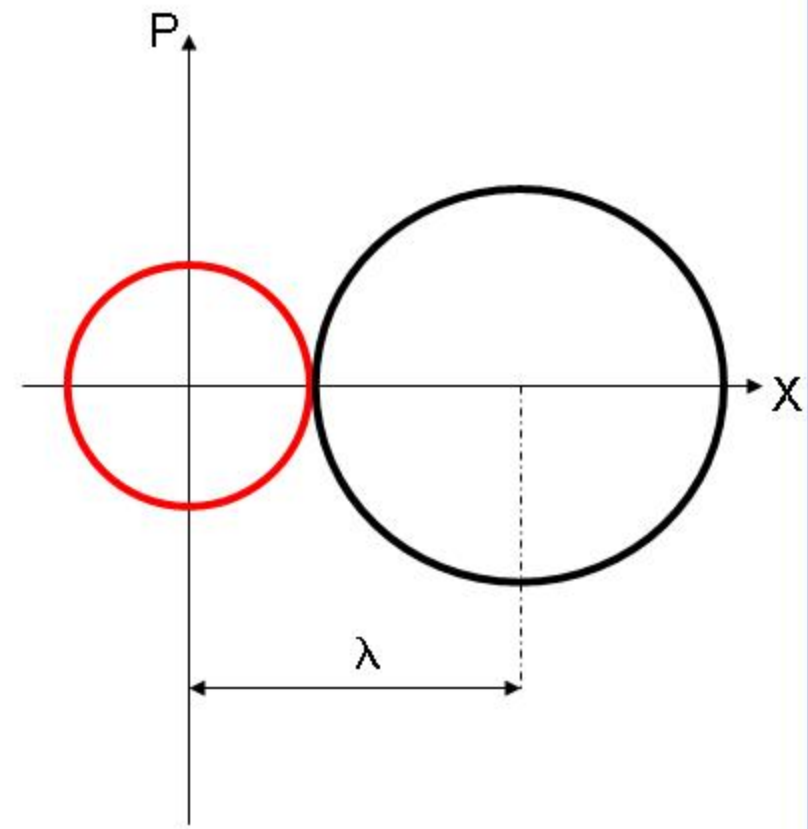
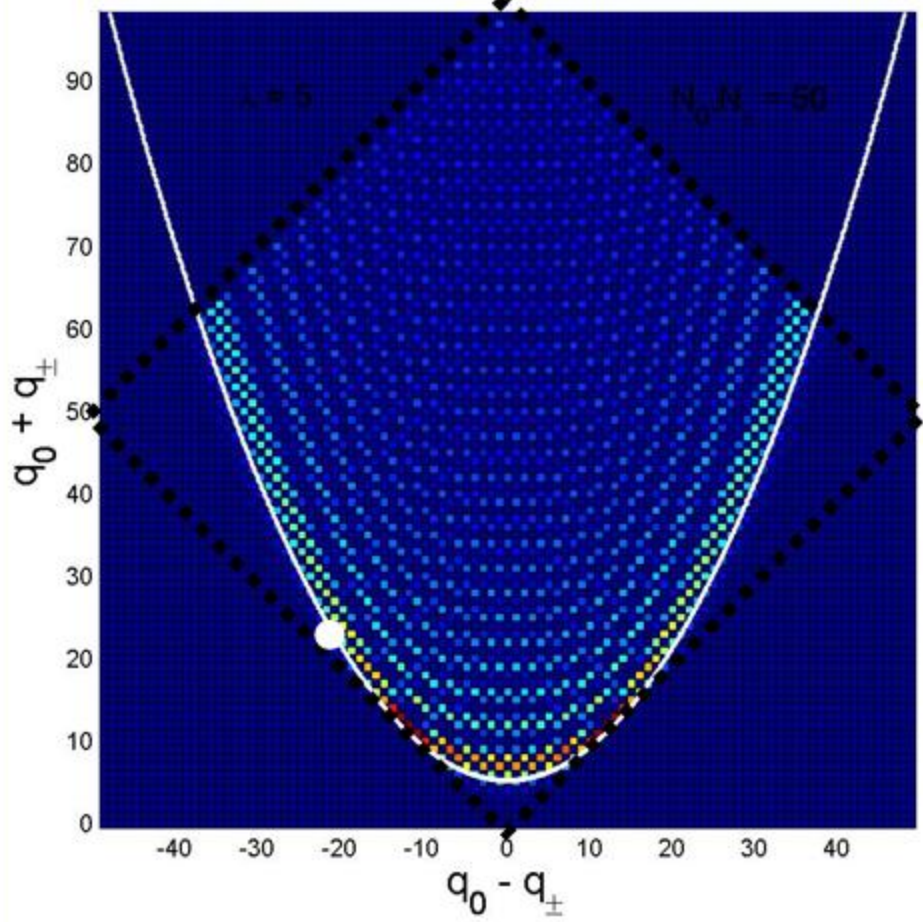
— State +  
— State 0





# The Franck-Condon parabola

Franck Condon rates



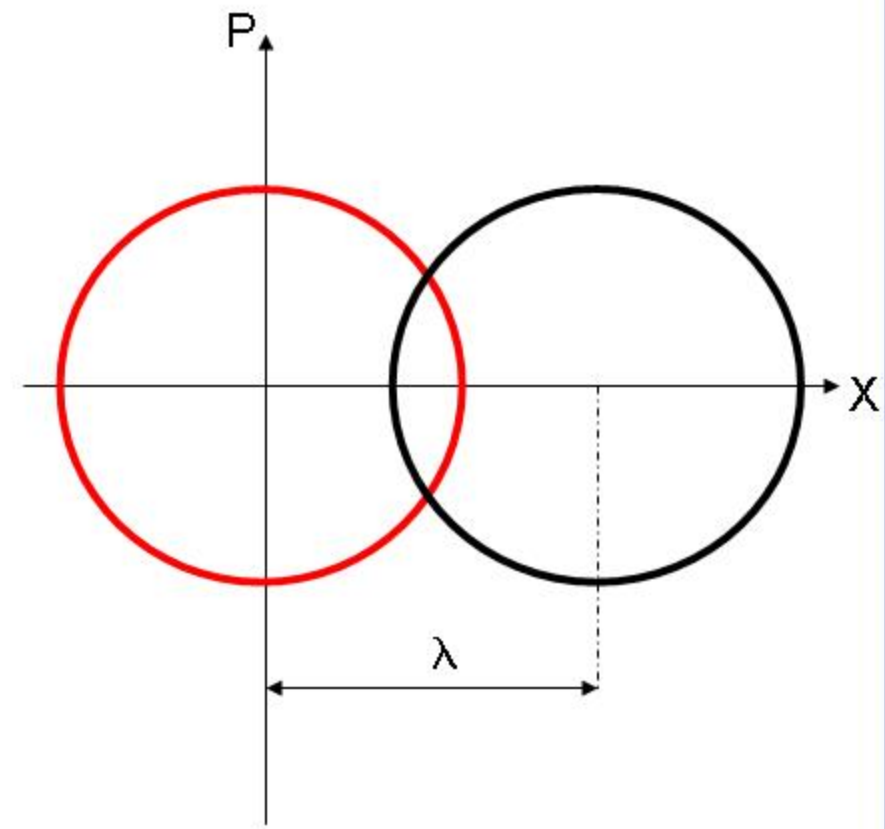
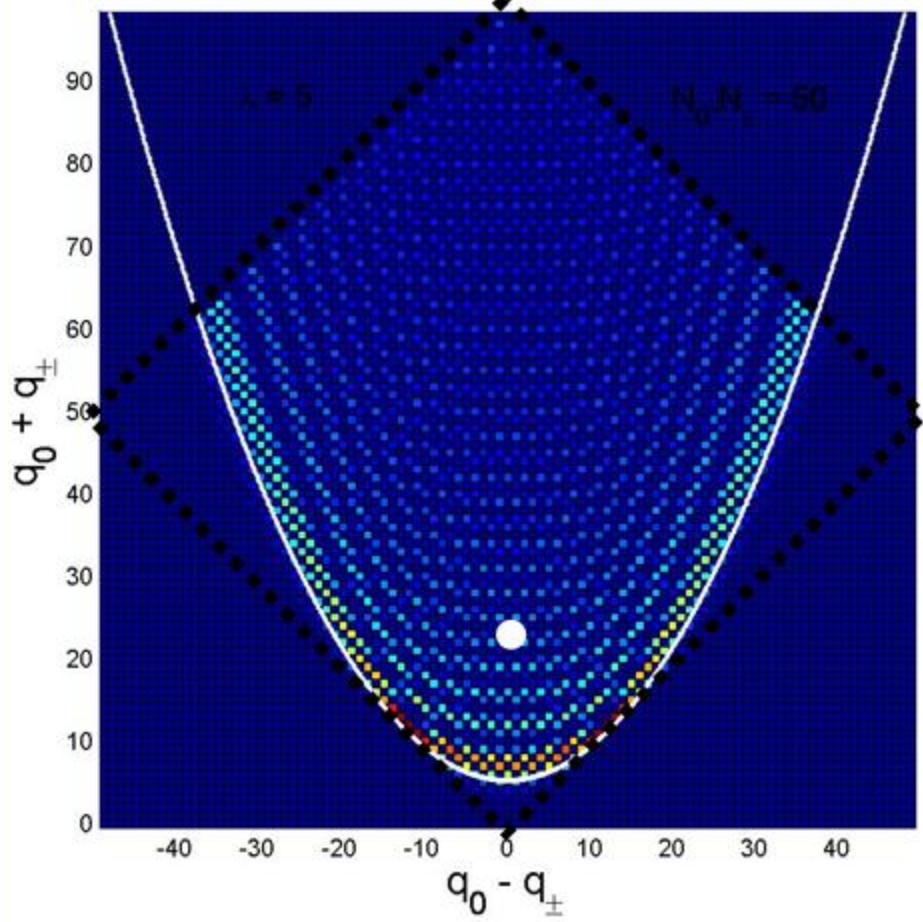
— State +  
— State 0





# The Franck-Condon parabola

Franck Condon rates



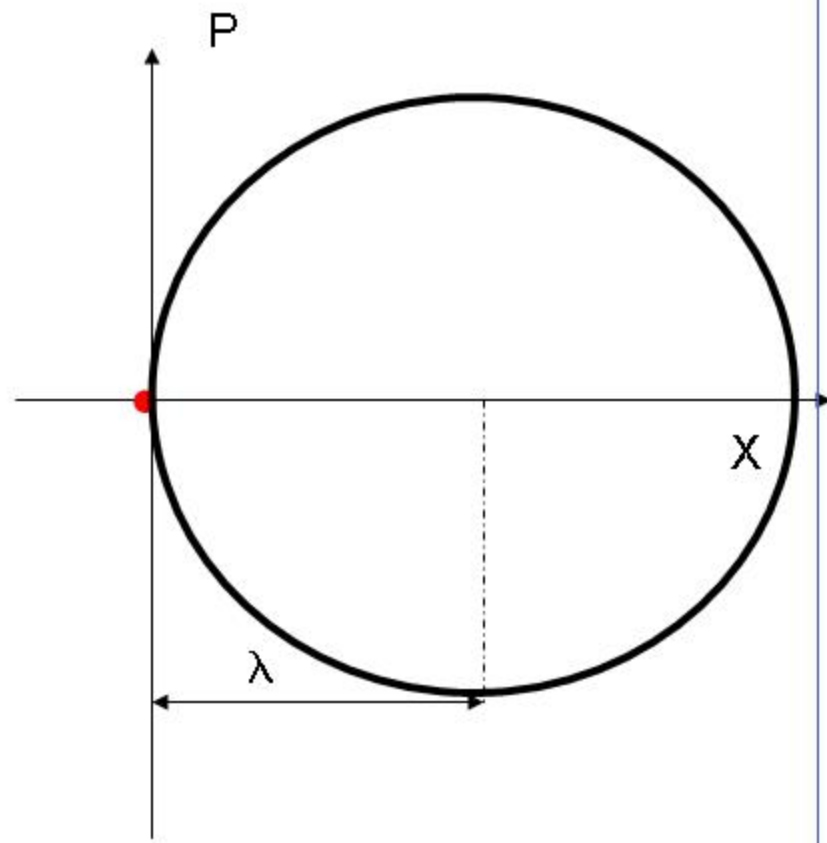
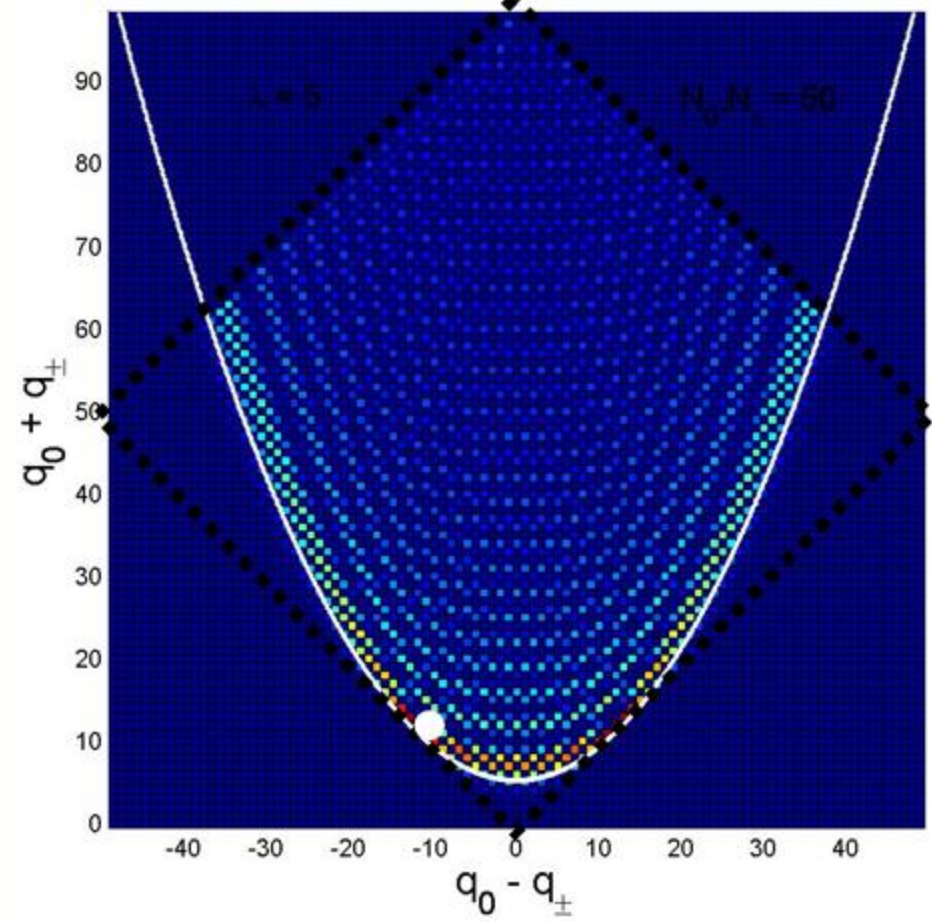
— State +  
— State 0





# The Franck-Condon parabola

Franck Condon rates



— State +  
— State 0





# The current calculation

1. Computation of the stationary reduced density matrix  $\sigma^{stat}$
2. Identification of the current operators:

$$\begin{aligned}
 I &= \frac{dQ}{dt} = \sum_m \dot{\sigma}_{11}^{m m} \\
 &= - \sum_{kl\alpha} \left( W_{kl}^\alpha \sigma_{11}^{lk} + \sigma_{11}^{kl} W_{lk}^\alpha \right) + \sum_{kn'\tau'n\tau\alpha} \left( R_{kn'n'\tau'n\tau}^\alpha + R_{kn'n\tau'n'\tau'}^\alpha \right) \sigma_{00}^{n'\tau'n\tau} \\
 &= - 2\text{Tr}_{Mech}\{W_1^L \sigma_{11}\} + 2\text{Tr}_{Mech}\{W_0^L \sigma_{00}\} - 2\text{Tr}_{Mech}\{W_1^R \sigma_{11}\} + 2\text{Tr}_{Mech}\{W_0^R \sigma_{00}\} \\
 &= \langle \hat{I}_L \rangle - \langle \hat{I}_R \rangle
 \end{aligned}$$

3. Calculation of the stationary current:

$$I_L^{stat} = \text{Tr}_{Mech}\{\sigma^{stat} \hat{I}_L\}$$

$$I_R^{stat} = \text{Tr}_{Mech}\{\sigma^{stat} \hat{I}_R\}$$





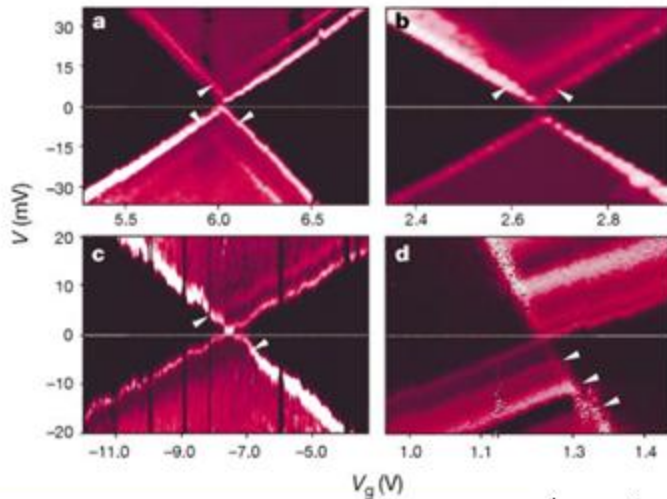


# Stability diagrams

- Plot of differential conductance as a function of bias and gate voltage:

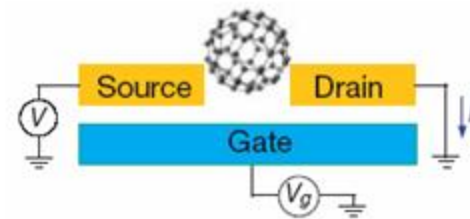
$$G = \frac{dI}{dV_b}(V_b, V_g)$$

- Helpful representation of tunneling spectroscopy:



## Nanomechanical oscillations in a single-C<sub>60</sub> transistor

Hongkun Park<sup>†,§</sup>, Jiwoong Park<sup>†</sup>, Andrew K. L. Lim<sup>\*</sup>, Erik H. Anderson<sup>‡</sup>, A. Paul Alivisatos<sup>†,‡</sup> & Paul L. McEuen<sup>†,‡</sup>

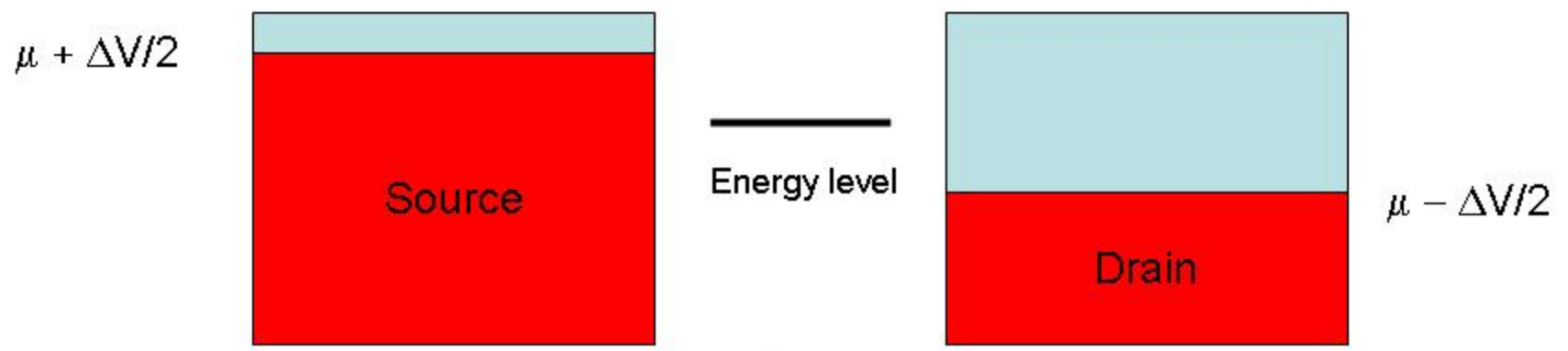
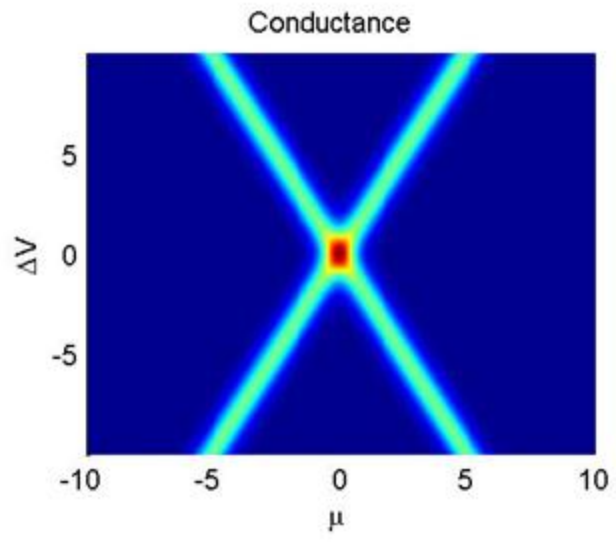
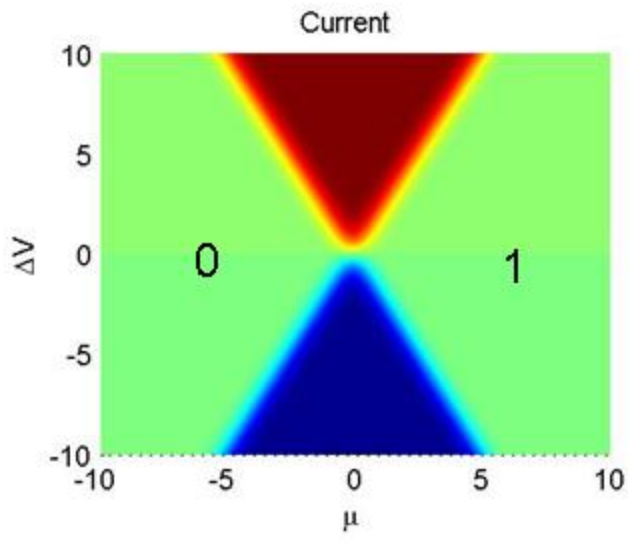


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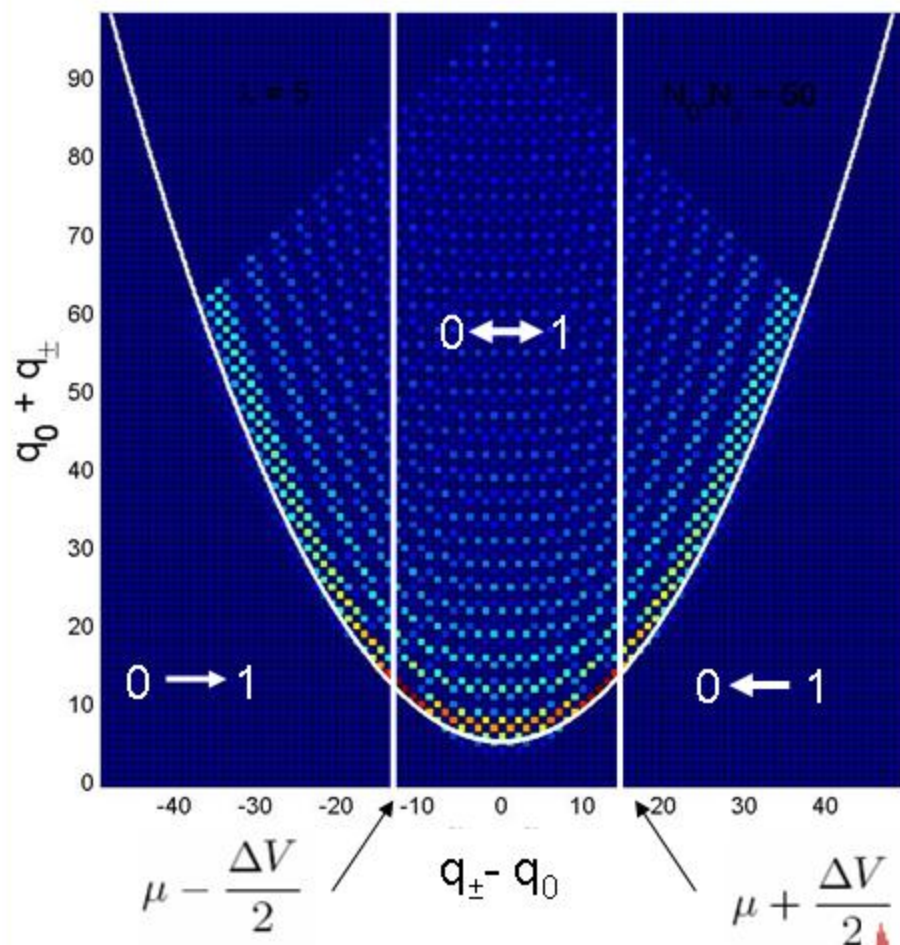
# Single level





# Franck-Condon parabola (II)

Franck Condon rates



Incoming electrons



$$\mu_L = \mu + \frac{\Delta V}{2} \geq \epsilon_1 - \epsilon_0 + (q_{\pm} - q_0)\hbar\omega$$

$$\mu_R = \mu - \frac{\Delta V}{2} \leq \epsilon_1 - \epsilon_0 + (q_{\pm} - q_0)\hbar\omega$$

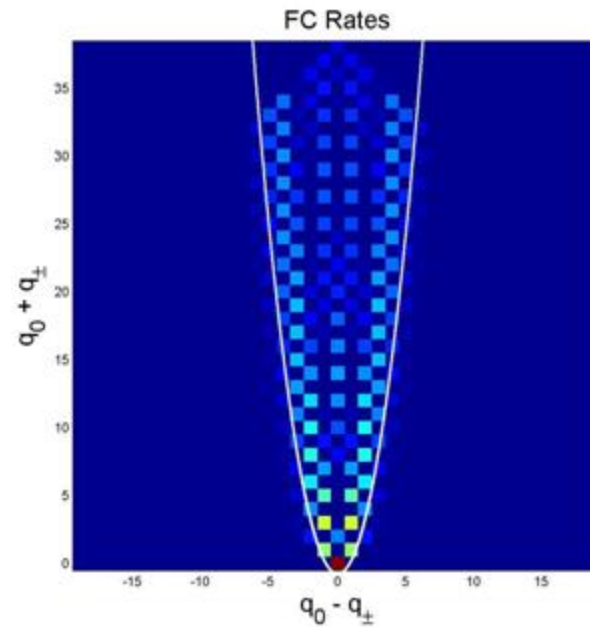
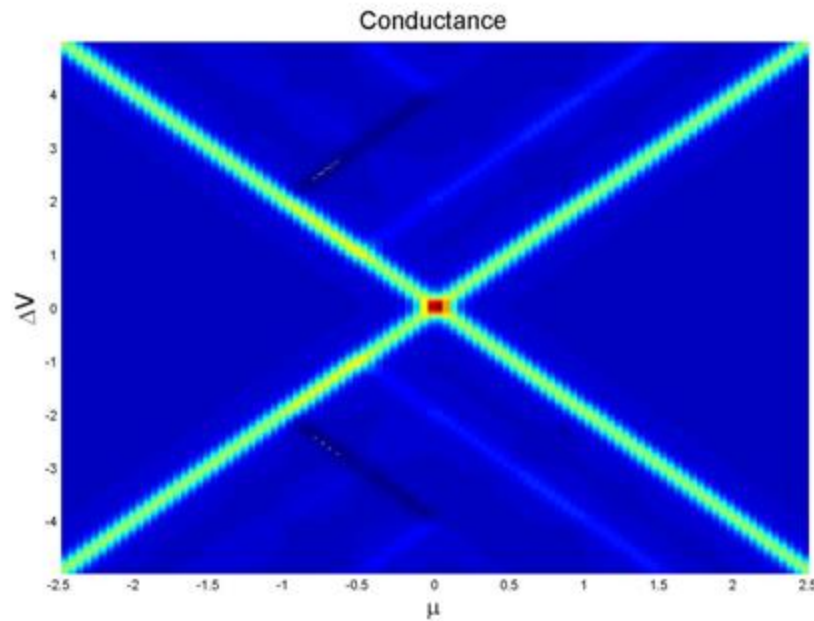


Outgoing electrons





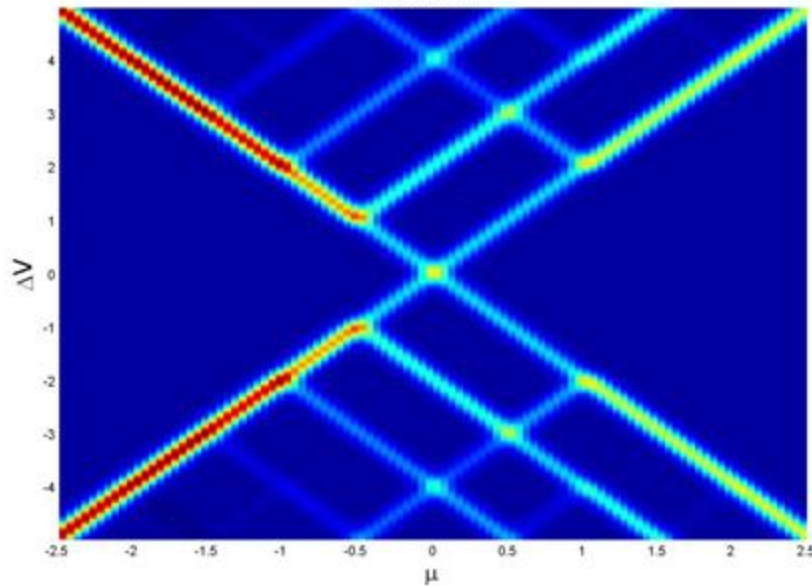
# Weak coupling $\lambda = 1$



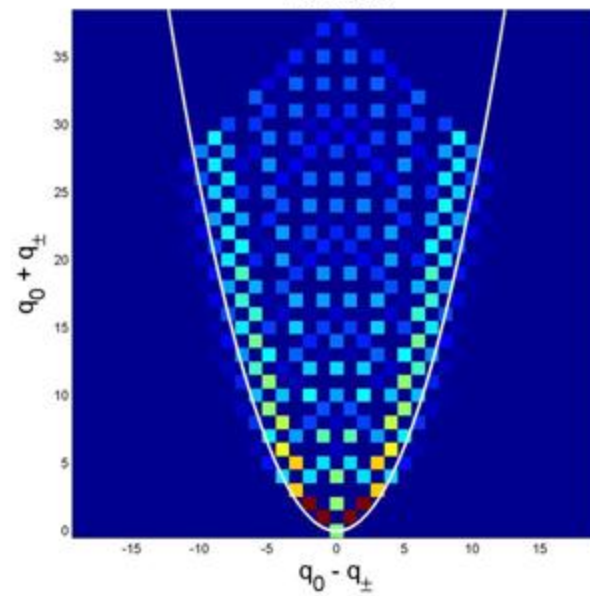


$$\lambda = 2$$

Conductance

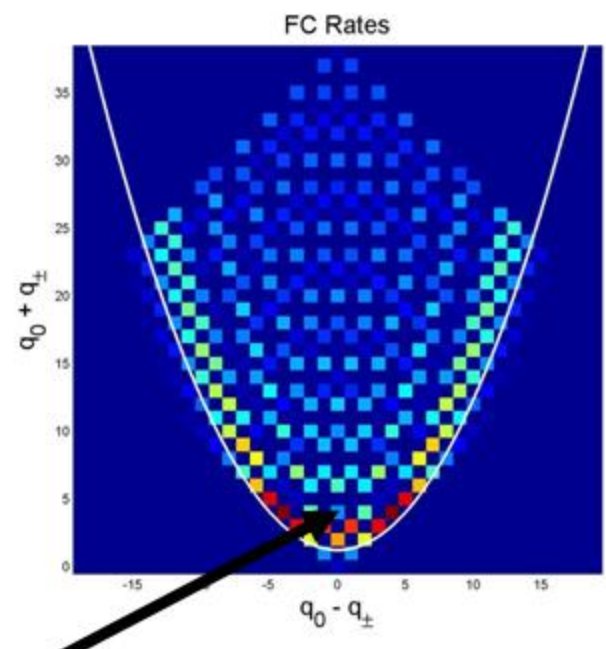
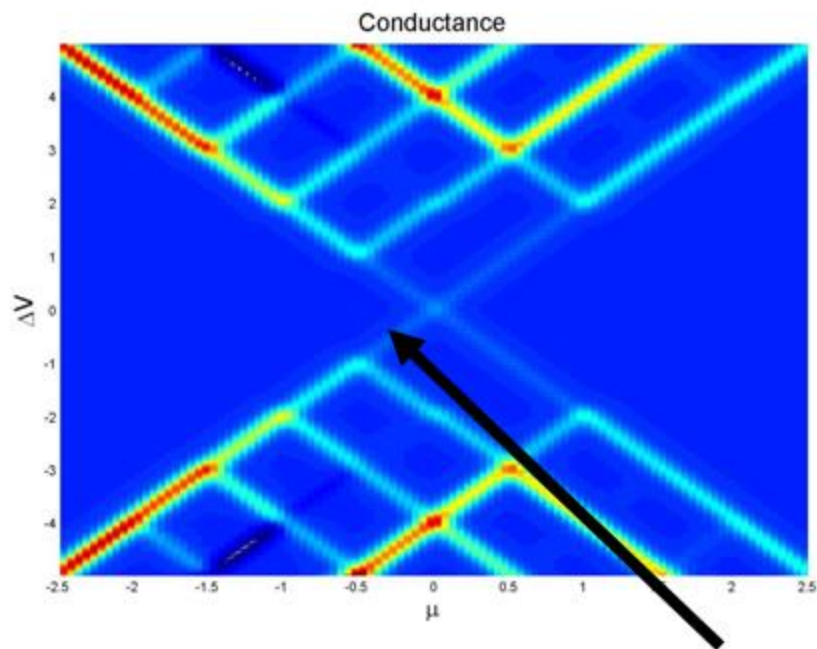


FC Rates





$$\lambda = 3$$

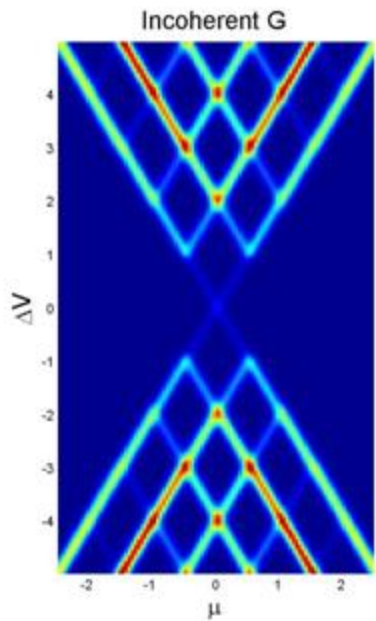
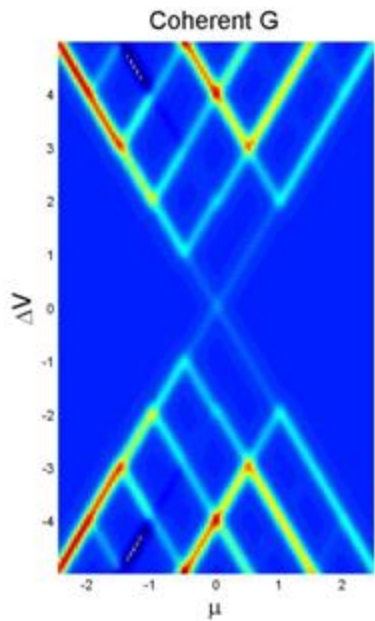


**Franck Condon Blockade**

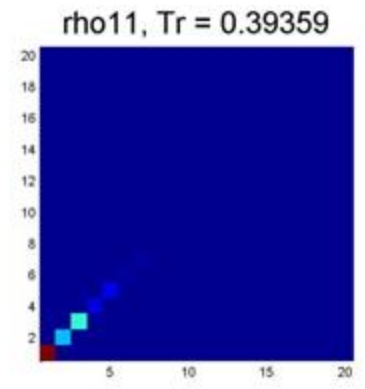
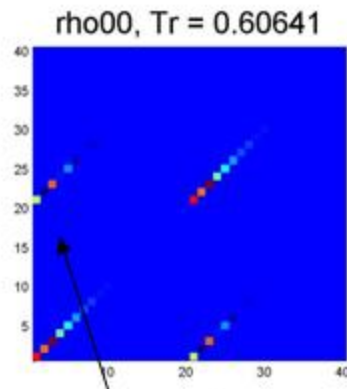




# Coherences...examples



$$\lambda = 1 \quad \Delta V = 5, \quad \mu = -1$$



Conductance evaluated neglecting coherences

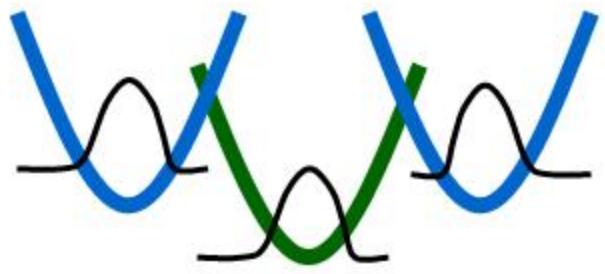
coherences



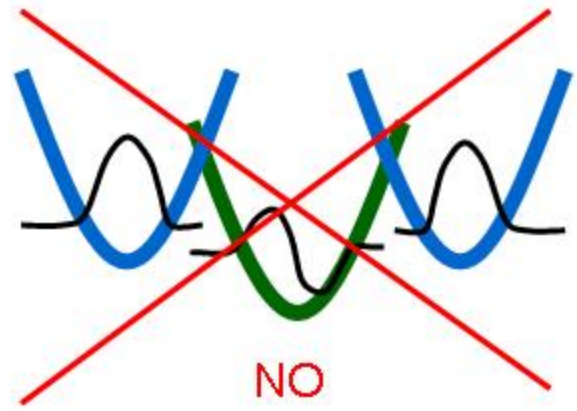


# Coherences...proof

- Tunneling **preserves parity** of the mechanical state
- In the even and odd sector degeneracies are lifted and a the stationary reduced density matrix is **diagonal**



YES



NO

- Franck-Condon coefficients are different in the even and odd sectors and give **different stationary solutions**
- Thus in the  $+ / -$  basis we must have coherences







# Summary

- We derived an **effective Hamiltonian** for the biphenyl transistor
- We obtained the description of the electromechanical dynamics in terms of a **GME**
- We observed the **Franck-Condon blockade** and gave an interpretation in terms of the Franck-Condon rates
- We observed the relevance of the **mechanical coherences** in the transport mechanism





# Still (a lot) to do!

- Understand the **anionic potential surface** in terms of the PPP Hamiltonian
- Understand the role of mechanical **coherences** in transport through biphenyl
- Use **realistic values** for the electron-vibron coupling in biphenyl ( $\lambda \sim 50$ )
- Investigate the role of the **mechanical bath**

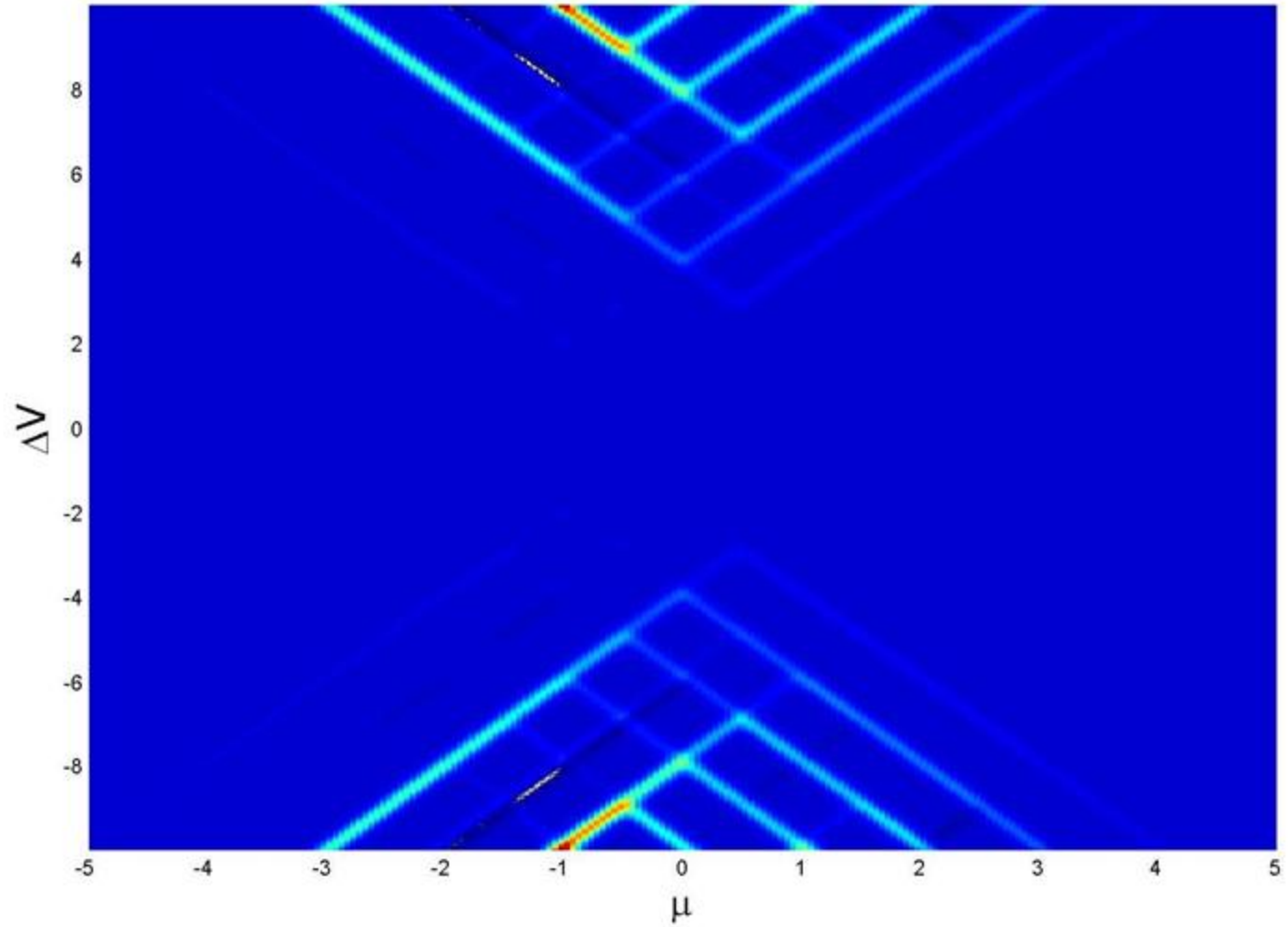
*Thanks for your attention!*





$$\lambda = 5$$

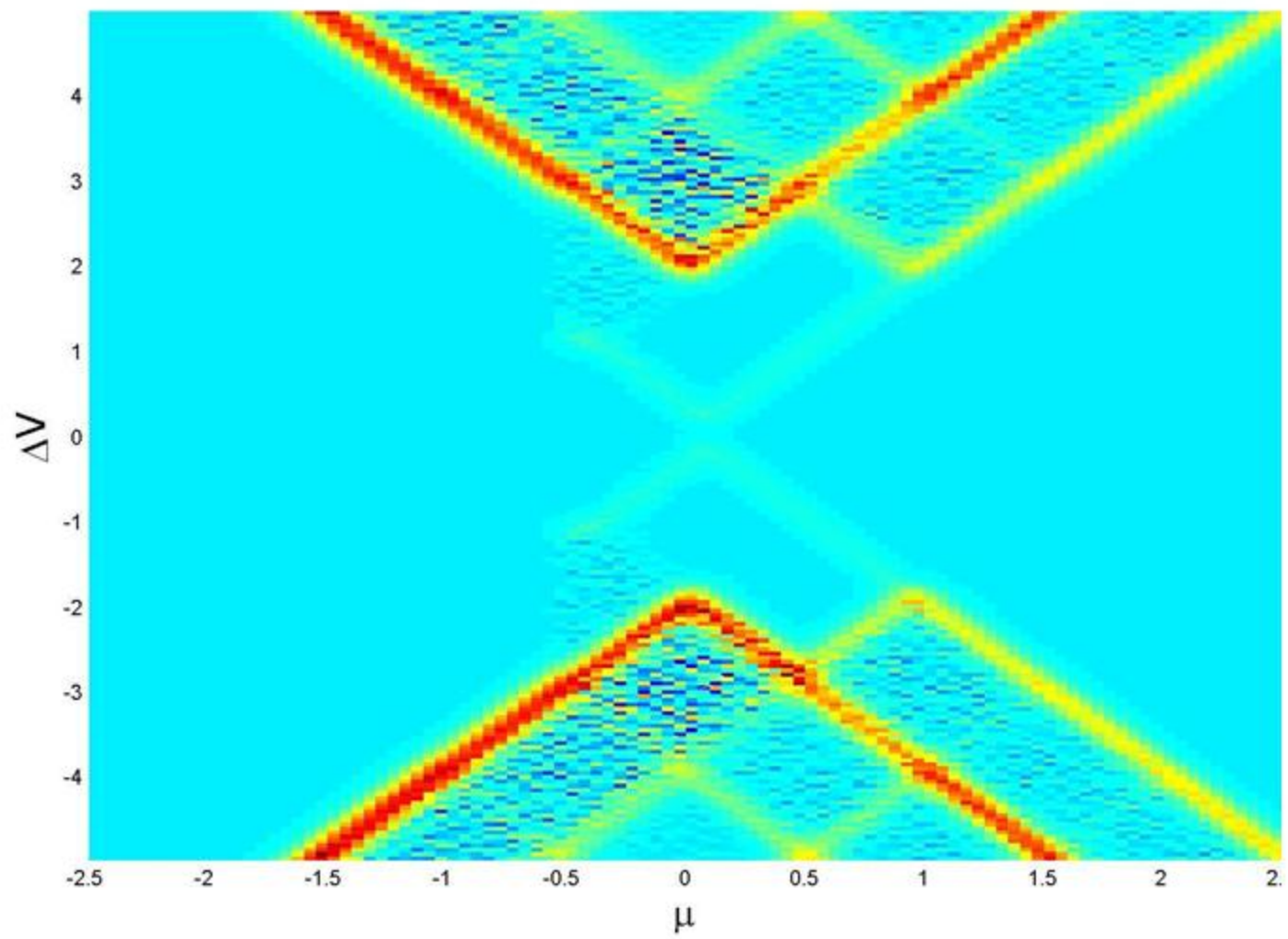
Conductance





# Bath ( $\lambda = 3$ )

Conductance





# Lambda scaling

