Fano stability diagram of a symmetric triple quantum dot

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GRK 1570

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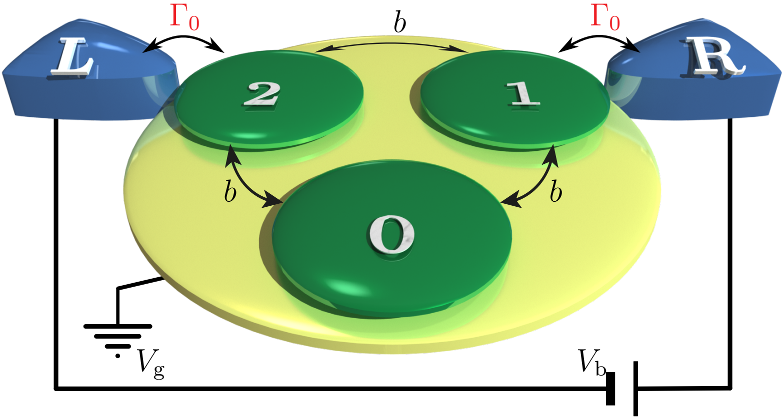
Triangular triple quantum dot



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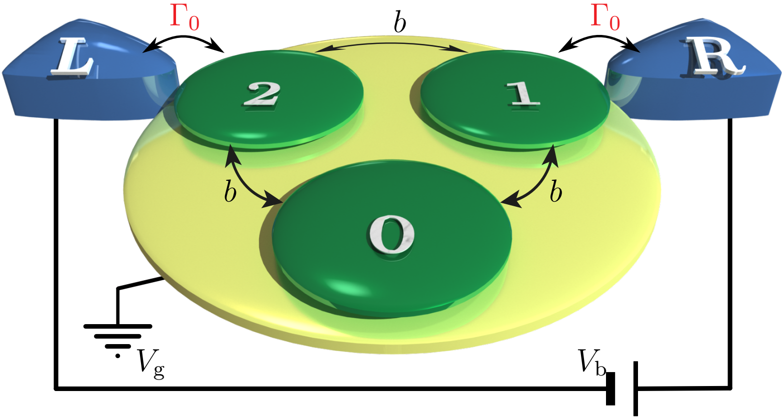
Triangular triple quantum dot



$$\hat{H} = \hat{H}_{\text{TQD}} + \hat{H}_{\text{res}} + \hat{H}_{\text{tun}}$$

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Motivation

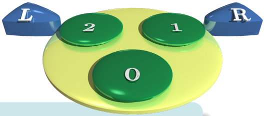
Why triangular triple quantum dot?

Smallest quantum dot system which shows

- orbitally induced interference [1]
- analytic, non trivial many-body states

[1] A. Donarini et al. - PRB **82**, 125451 (2010)

Motivation



Why triangular triple quantum dot?

Smallest quantum dot system which shows

- orbitally induced interference [1]
- analytic, non trivial many-body states

Why noise?

- holds information about interplay of statistics, geometry and interactions
- unravels underlying bunching mechanisms

[1] A. Donarini et al. - PRB **82**, 125451 (2010)

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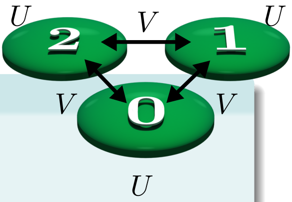
Model



Extended Hubbard model

$$\hat{H}_{\mathsf{TQD}} = \xi \sum_{i\sigma} n_{i\sigma} + b \sum_{i\neq j,\sigma} d^{\dagger}_{j\sigma} d_{i\sigma} + U \sum_{i} \left(n_{i\uparrow} - \frac{1}{2}\right)\left(n_{i\downarrow} - \frac{1}{2}\right) + V \sum_{i<j} (n_i - 1)(n_j - 1)$$

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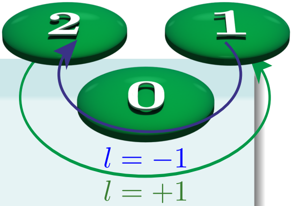
Model



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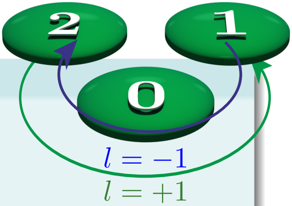
- single particle part is diagonal in angular momentum basis

Model



Extended Hubbard model

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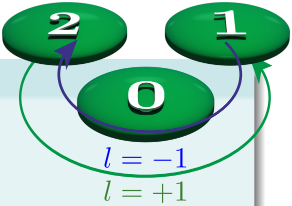
- single particle part is diagonal in angular momentum basis
- many-body states are fully characterized by $|N, E; S, S_z, L_z\rangle$

Model



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- single particle part is diagonal in angular momentum basis
- many-body states are fully characterized by $|N, E; S, S_z, L_z\rangle$
- analytical eigenstates

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Eigenstates

N	Eigenenergy	S	S_z	L_z	Eigenstate in the basis $\{\lvert n_{0\uparrow}, n_{1\uparrow}, n_{-1\uparrow}; n_{0\downarrow}, n_{1\downarrow}, n_{-1\downarrow}\rangle\}$
0	$E_0 = 0$	0	0	0	$\lvert 000, 000\rangle$
1	$E_{1_0} = \xi - \frac{U}{2} - 2V + 2b$	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\lvert 000, 100\rangle$
			$\frac{1}{2}$		$\lvert 100, 000\rangle$
	$E_{1_1} = \xi - \frac{U}{2} - 2V - b$	$\frac{1}{2}$	$-\frac{1}{2}$	-1	$\lvert 000, 001\rangle$
				1	$\lvert 000, 010\rangle$
			$\frac{1}{2}$	-1	$\lvert 001, 000\rangle$
				1	$\lvert 010, 000\rangle$
2	$E_{2_0} = 2\xi - U - 3V + b + \frac{U-V}{2} - s_{-2}$	0	0	0	$\cos(\phi_{-2})\lvert 100, 100\rangle - \sin(\phi_{-2})\frac{1}{\sqrt{2}}\left(\lvert 010, 001\rangle + \lvert 001, 010\rangle\right)$
	$E_{2_1} = 2\xi - U - 3V + b$	1	-1	-1	$\lvert 000, 101\rangle$
				1	$\lvert 000, 110\rangle$
			0	-1	$\frac{1}{\sqrt{2}}\left(\lvert 100, 001\rangle - \lvert 001, 100\rangle\right)$
				1	$\frac{1}{\sqrt{2}}\left(\lvert 100, 010\rangle - \lvert 010, 100\rangle\right)$
			1	-1	$\lvert 101, 000\rangle$
				1	$\lvert 110, 000\rangle$
	$E_{2_2} = 2\xi - U - 3V - \frac{b}{2} + \frac{U-V}{2} - s_1$	0	0	-1	$\cos(\phi_1)\lvert 010, 010\rangle - \sin(\phi_1)\frac{1}{\sqrt{2}}\left(\lvert 100, 001\rangle + \lvert 001, 100\rangle\right)$
				1	$\cos(\phi_1)\lvert 001, 001\rangle - \sin(\phi_1)\frac{1}{\sqrt{2}}\left(\lvert 100, 010\rangle + \lvert 010, 100\rangle\right)$
	$E_{2_3} = 2\xi - U - 3V - 2b$	1	-1	0	$\lvert 000, 011\rangle$
			0		$\frac{1}{\sqrt{2}}\left(\lvert 010, 001\rangle - \lvert 001, 010\rangle\right)$
			1		$\lvert 011, 000\rangle$
	$E_{2_4} = 2\xi - U - 3V - \frac{b}{2} + \frac{U-V}{2} + s_1$	0	0	-1	$\sin(\phi_1)\lvert 010, 010\rangle + \cos(\phi_1)\frac{1}{\sqrt{2}}\left(\lvert 100, 001\rangle + \lvert 001, 100\rangle\right)$
				1	$\sin(\phi_1)\lvert 001, 001\rangle + \cos(\phi_1)\frac{1}{\sqrt{2}}\left(\lvert 100, 010\rangle + \lvert 010, 100\rangle\right)$
	$E_{2_5} = 2\xi + b - U - 3V + \frac{U-V}{2} + s_{-2}$	0	0	0	$\sin(\phi_{-2})\lvert 100, 100\rangle + \cos(\phi_{-2})\frac{1}{\sqrt{2}}\left(\lvert 010, 001\rangle + \lvert 001, 010\rangle\right)$
	$E_{3_0} = 3\xi - \frac{3}{2}U - 3V + \frac{2}{3}\left(U - V\right)\left[1 - \lambda_0/(2\lvert a\rvert)\right]$	$\frac{1}{2}$	$-\frac{1}{2}$	-1	$v_{0,1}\lvert 100, 101\rangle - v_{0,0}\lvert 010, 110\rangle - v_{0,-1}\lvert 001, 011\rangle$
				1	$v_{0,1}\lvert 100, 110\rangle + v_{0,0}\lvert 001, 101\rangle - v_{0,-1}\lvert 010, 011\rangle$
			$\frac{1}{2}$	-1	$v_{0,1}\lvert 101, 100\rangle - v_{0,0}\lvert 110, 010\rangle - v_{0,-1}\lvert 011, 001\rangle$
				1	$v_{0,1}\lvert 110, 100\rangle - v_{0,0}\lvert 101, 001\rangle + v_{0,-1}\lvert 011, 010\rangle$
	$E_{3_1} = 3\xi - \frac{3}{2}U - 3V$	$\frac{3}{2}$	$-\frac{3}{2}$	0	$\lvert 000, 111\rangle$
			$-\frac{1}{2}$		$\frac{1}{\sqrt{3}}\left(\lvert 001, 110\rangle - \lvert 010, 101\rangle + \lvert 100, 011\rangle\right)$
			$\frac{1}{2}$		$\frac{1}{\sqrt{3}}\left(\lvert 011, 100\rangle - \lvert 101, 010\rangle + \lvert 110, 001\rangle\right)$

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	$E_{2_2} = 2\xi - U - 3V - \frac{b}{2} + \frac{U-V}{2} - s_1$	0	0	-1	$\cos(\phi_1)\lvert 010, 010\rangle - \sin(\phi_1)\frac{1}{\sqrt{2}}(\lvert 100, 001\rangle + \lvert 001, 100\rangle)$
				1	$\cos(\phi_1)\lvert 001, 001\rangle - \sin(\phi_1)\frac{1}{\sqrt{2}}(\lvert 100, 010\rangle + \lvert 010, 100\rangle)$
	$E_{2_3} = 2\xi - U - 3V - 2b$	1	-1	0	$\lvert 000, 011\rangle$
			0		$\frac{1}{\sqrt{2}}(\lvert 010, 001\rangle - \lvert 001, 010\rangle)$
			1		$\lvert 011, 000\rangle$
	$E_{2_4} = 2\xi - U - 3V - \frac{b}{2} + \frac{U-V}{2} + s_1$	0	0	-1	$\sin(\phi_1)\lvert 010, 010\rangle + \cos(\phi_1)\frac{1}{\sqrt{2}}(\lvert 100, 001\rangle + \lvert 001, 100\rangle)$
				1	$\sin(\phi_1)\lvert 001, 001\rangle + \cos(\phi_1)\frac{1}{\sqrt{2}}(\lvert 100, 010\rangle + \lvert 010, 100\rangle)$
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				1	$v_{0,1}\lvert 100, 110\rangle + v_{0,0}\lvert 001, 101\rangle - v_{0,-1}\lvert 010, 011\rangle$
			$\frac{1}{2}$	-1	$v_{0,1}\lvert 101, 100\rangle - v_{0,0}\lvert 110, 010\rangle - v_{0,-1}\lvert 011, 001\rangle$
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			$\frac{1}{2}$		$\frac{1}{\sqrt{3}}(\lvert 011, 100\rangle - \lvert 101, 010\rangle + \lvert 110, 001\rangle)$
			$\frac{3}{2}$		$\lvert 111, 000\rangle$

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Method

Generalized reduced density matrix

$$\rho_\chi = \mathrm{tr}_{\mathsf{res}} \left\{ e^{i\chi N} \rho^{\mathrm{tot}} \right\} = \rho + \sum_{k=1}^{\infty} \frac{(i\chi)^k}{k!} \mathcal{F}_k$$

Generalized master equation

$$\dot{\rho}_\chi = \mathcal{L}\rho_\chi + \left(e^{i\chi} - 1 \right) \mathcal{J}^+ \rho_\chi + \left(e^{-i\chi} - 1 \right) \mathcal{J}^- \rho_\chi$$

Liouvillian

$$\mathcal{L}\rho = -\frac{i}{\hbar} \left[\hat{H}_{\mathsf{TQD}} + \hat{H}_{\mathsf{LS}}, \rho \right] + \mathcal{L}_{\mathsf{tun}} \rho$$

- orbital degeneracies require inclusion of coherences

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Method

Stationary solution[2]

$$\mathcal{L}\rho^\infty = 0$$

$$\mathcal{L}\mathcal{F}_{1\perp}^\infty = \left(-eI - \mathcal{J}^+ + \mathcal{J}^-\right)\rho^\infty$$

with $\mathcal{F}_{1\perp} = (1 - \rho^\infty \mathrm{tr}_{\mathsf{TQD}})\mathcal{F}_1$

Current, noise and Fano factor

$$I = -e\partial_t \langle N \rangle = -e\mathrm{tr}_{\mathsf{TQD}}\left\{\left(\mathcal{J}^+ - \mathcal{J}^-\right)\rho^\infty\right\}$$

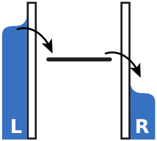
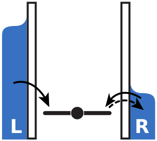
$$S = e^2\partial_t\left(\langle N^2\rangle - \langle N\rangle^2\right)$$

$$= e^2\mathrm{tr}_{\mathsf{TQD}}\left\{2\left(\mathcal{J}^+ - \mathcal{J}^-\right)\mathcal{F}_{1\perp}^\infty + \left(\mathcal{J}^+ + \mathcal{J}^-\right)\rho^\infty\right\}$$

$$F = \frac{S}{e|I|}$$

[2] F. Kaiser and S. Kohler - Ann. Phys. **16**, 702 (2007)

The Fano factor

$F < 1$	$F = 1$	$F > 1$
Anti-bunching “fermion like”	Independent tunneling events	Bunching “boson like”
		this talk
$\Downarrow$	$\Downarrow$	$\Downarrow$
Sub-Poissonian Noise	Poissonian Noise	Super-Poissonian Noise

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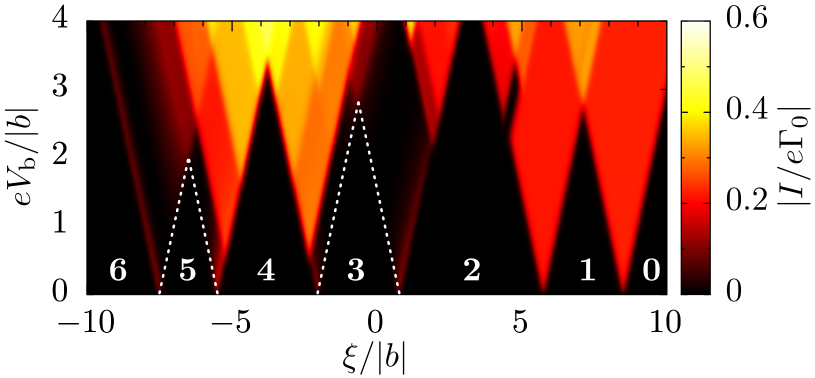
Stability diagram

$U = 5|b|$, $V = 2|b|$, $k_B T = 0.002|b|$, $k_B T = 20\Gamma$ and $b < 0$



- interference blockade → dark states

[1]A. Donarini et al. - PRB **82**, 125451 (2010)

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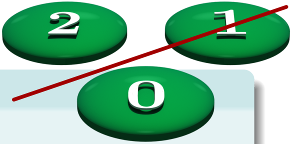
Dark states



Dark states

$$|N,\alpha_i;\mathrm{DS}\rangle = \frac{1}{\sqrt{2}}\left[e^{i\frac{2\pi}{3}}|N,\alpha_i,L_z=1\rangle - e^{-i\frac{2\pi}{3}}|N,\alpha_i,L_z=-1\rangle\right]$$

- simple expression in angular momentum representation
- every level with angular momentum degeneracy can form a DS
- DS is antisymmetric with respect to σ_{v1}

fulfills [3]

$$\langle N-1,\alpha_i;L_z=0|d_{1\sigma}|N,\alpha_j;\mathrm{DS}\rangle = 0$$

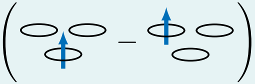
[3] T. Kostyrko and B. Bułka - PRB **79**, 075310 (2009)

Dark states

Example DSs in position basis:

one-particle first excited state with $S_z = 1/2$ [4]

$$|1, \alpha_1; \mathrm{DS}\rangle = \frac{1}{\sqrt{2}}$$



[4] B. Michaelis *et al.* - EPL **73**, 677 (2006)

Dark states

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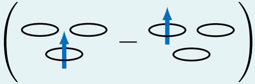
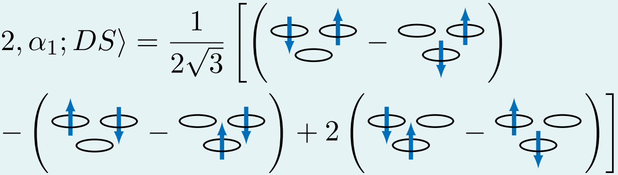


two-particle first excited state with $S_z = 0$

$$|2, \alpha_1; DS\rangle = \frac{1}{2\sqrt{3}}$$



[4] B. Michaelis *et al.* - EPL **73**, 677 (2006)

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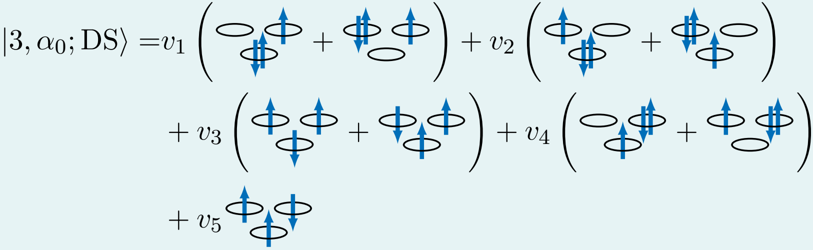
Dark states

three-particle ground state with $S_z = 1/2$



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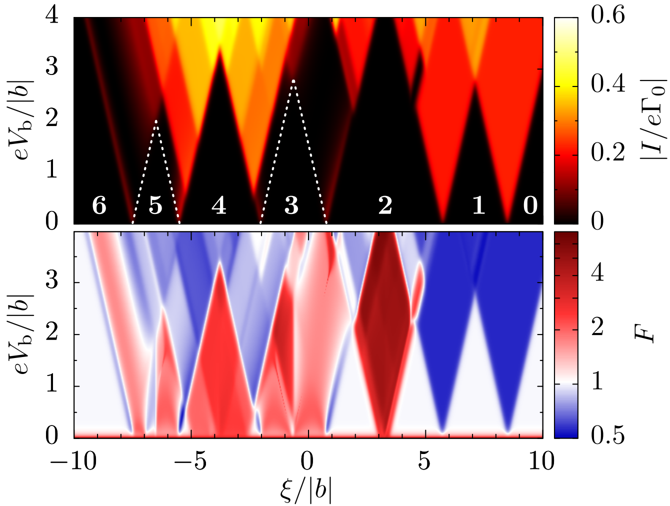
Stability diagram for I and F



$U = 5|b|$, $V = 2|b|$, $k_B T = 0.002|b|$, $k_B T = 20\Gamma$ and $b < 0$

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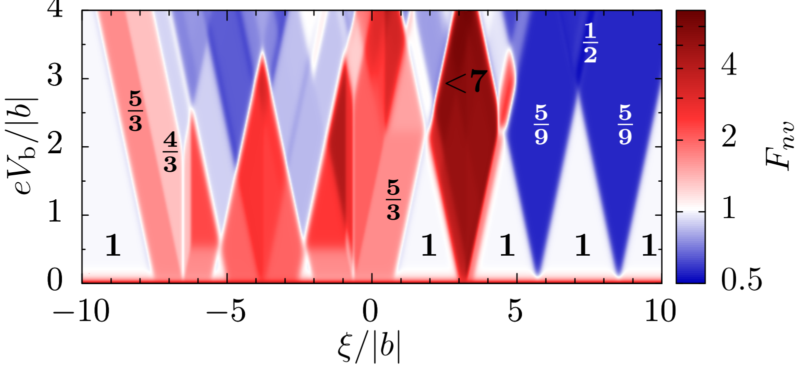
Stability diagram without principle parts

Principal parts (H_{LS}) blur everything $\rightarrow$ solve without



- clear polygons
- super-Poissonian noise ($F > 1$) indicates blocking

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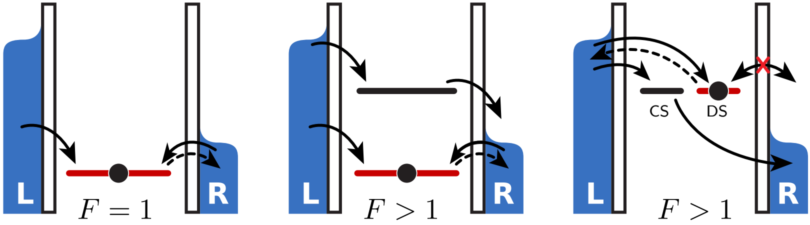
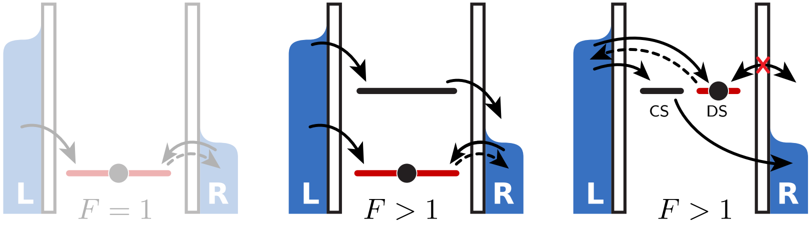
Blockade mechanisms



Coulomb blockade   Channel blockade[5,6]   Interference blockade

[5] W. Belzig - PRB **71**, 161301(R) (2005)
[6] C.W. Groth et al. - PRB **74**, 125315 (2006)

Blockade mechanisms



Coulomb blockade Channel blockade[5,6] Interference blockade

Effective slow+fast channel model provides Fano factor

$$F_{nv} = 1 + \frac{2\Gamma_L^f}{\Gamma_L^s + \Gamma_R^s},$$

[5] W. Belzig - PRB **71**, 161301(R) (2005)
[6] C.W. Groth et al. - PRB **74**, 125315 (2006)

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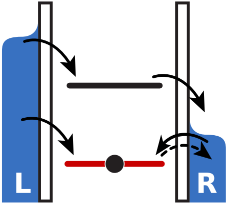
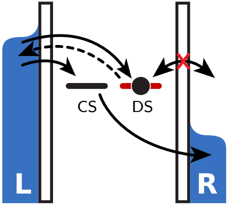
Blockade mechanisms at the $2_0 \leftrightarrow 3_0$ resonance

$$F_{nv} = 1 + \frac{2\Gamma_L^f}{\Gamma_L^s + \Gamma_R^s},$$



?

$\Leftrightarrow$



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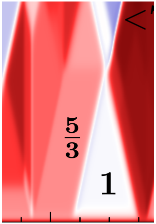

Blockade mechanisms at the $2_0 \leftrightarrow 3_0$ resonance

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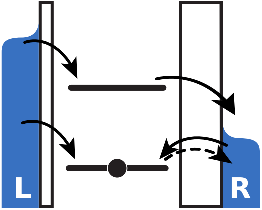
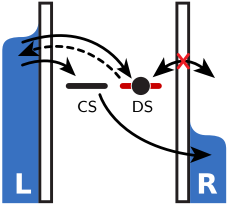


$$\Gamma_R^s \approx 0$$

$$\Leftrightarrow$$



$$\Gamma_L^f = \Gamma_L^s$$

$$\downarrow$$

$$F_{nv} = 3$$

$$\Gamma_L^f \neq \Gamma_L^s \quad ?$$

$$\downarrow$$

$$F_{nv} = \frac{5}{3}$$

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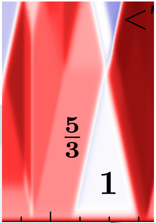

Fingerprints of interference at $2_0 \leftrightarrow 3_0$

Dynamics for $\mu_L \gg \mu_R$

$$\dot{\rho}_3 = 0 = 2\Gamma \mathcal{R}_L \rho_2 - \frac{\Gamma}{2} \left\{ \mathcal{R}_R, \rho_3 \right\}$$

$\Gamma_\alpha = \Gamma \mathcal{R}_\alpha$ and the rate matrices in

angular momentum basis

$$\mathcal{R}_L = \begin{pmatrix} 1 & e^{i\frac{2\pi}{3}} \\ e^{-i\frac{2\pi}{3}} & 1 \end{pmatrix}$$

$$\mathcal{R}_R = \begin{pmatrix} 1 & e^{-i\frac{2\pi}{3}} \\ e^{i\frac{2\pi}{3}} & 1 \end{pmatrix}$$

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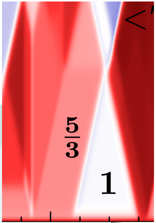

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dark state basis

$$\mathcal{R}_L = \begin{pmatrix} \frac{3}{2} & -i\frac{\sqrt{3}}{2} \\ i\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$$

$$\mathcal{R}_R = \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix}$$

$$F_{nv} = 1 + \frac{2\Gamma_L^f}{\Gamma_L^s + \Gamma_R^s}$$

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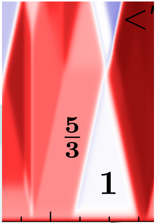

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$$F_{nv} = 1 + \frac{2\Gamma_L^f}{\Gamma_L^s + \Gamma_R^s} = 1 + \frac{2\frac{1}{2}}{\frac{3}{2} + 0} = \frac{5}{3}$$

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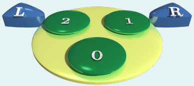
Conclusion

PRB **95**, 115133 (2017)

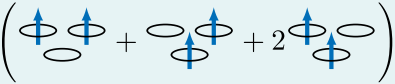
interference



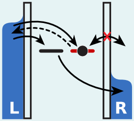
Interference occurs when energetically equivalent paths involving degenerate states contribute to the dynamics

dark states

$$|2, \alpha_1; DS\rangle = \frac{1}{\sqrt{6}}$$



fingerprints of interference

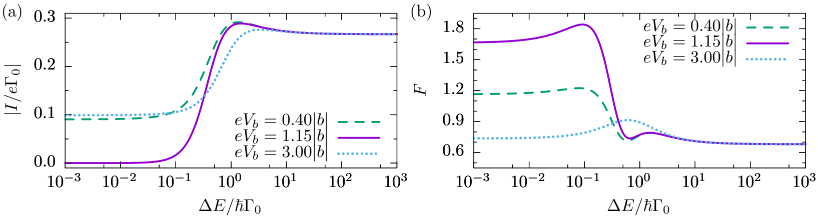


super-Poissonian Fano factors (e.g. $F = 5/3$) which indicate a characteristic bunching dynamics

Robustness

break angular momentum degeneracy

$$H_\Delta = -\Delta E \sigma_z / 2 \quad \Rightarrow \quad H_\Delta = \frac{1}{2}\begin{pmatrix} 0 & \Delta E \\ \Delta E & 0 \end{pmatrix}$$



$\Rightarrow$ stable up to perturbation of the order Γ_0

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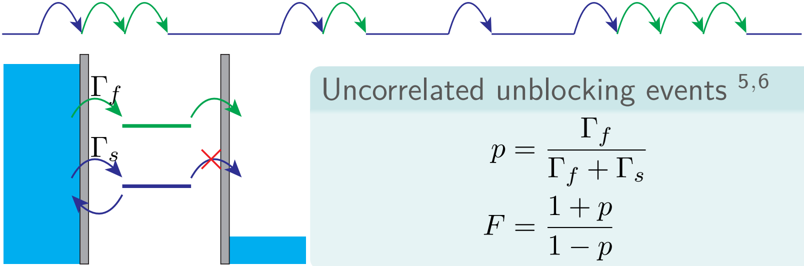
Super-Poissonian Fano factors



Uncorrelated unblocking events [5,6]

$$p = \frac{\Gamma_f}{\Gamma_f + \Gamma_s}$$

$$F = \frac{1+p}{1-p}$$

Two levels

- probability to enter the blocking state is $p = \frac{1}{2}$
- $F = 3$

[5] W. Belzig - PRB **71**, 161301(R) (2005)
[6] C.W. Groth et al. - PRB **74**, 125315 (2006)

Lamb shift

Hamiltonian

$$H_{\mathrm{LS}} = \hbar \sum_{\alpha} \omega_{\alpha} \mathcal{R}_{\alpha}$$

precession frequencies

The precession frequencies for the block $\rho^{N}(E^{*})$ with spin S is independent of S_z ($\omega_{\alpha,S_z} = \omega_{\alpha}$)

$$\omega_{\alpha} = \frac{\Gamma_{0\alpha}}{2\pi} \sum_{\tau,E} \langle N, \alpha^{*}, L_z | d_{0\tau} \mathcal{P}_{N+1,E} d_{0\tau}^{\dagger} | N, \alpha^{*}, -L_z \rangle p_{\alpha}\left(E - E^{*}\right) \\ + \langle N, \alpha^{*}, L_z | d_{0\tau}^{\dagger} \mathcal{P}_{N-1,E} d_{0\tau} | N, \alpha^{*}, -L_z \rangle p_{\alpha}\left(E^{*} - E\right)$$

- $\mathcal{P}_{NE} = \sum_{S_z,L_z} |N, E; S, S_z, L_z\rangle\langle N, E; S, S_z, L_z|$
- $p_{\alpha}\left(\Delta E\right) = -\operatorname{Re}\psi\left(\frac{1}{2} + i\frac{\Delta E - \mu_{\alpha}}{2\pi k_{\mathrm{B}} T}\right)$

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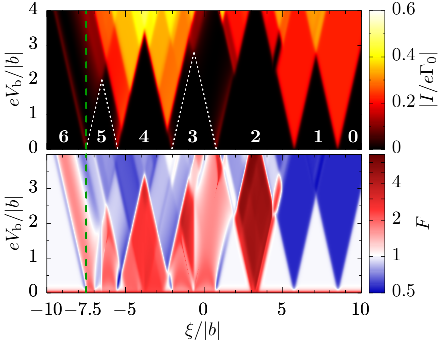
Lamb shift at the $5_0 \leftrightarrow 6$ resonance



master equation

$$0 = -\frac{i}{\hbar}\left[H_{\mathrm{LS}}, \rho_5\right] + 2\Gamma \mathcal{R}_R \rho_6 - \frac{\Gamma}{2}\left\{\mathcal{R}_L, \rho_5\right\},$$

$$0 = \Gamma \mathrm{Tr}_{\mathrm{TQD}}\left(\mathcal{R}_L \rho_5\right) - 4\Gamma\rho_6$$

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Lamb shift at the $5_0 \leftrightarrow 6$ resonance

$$\rho^\infty = \frac{1}{D}\begin{pmatrix} D - 3\omega_R^2 \\ 2\omega_R^2 \\ \omega_R^2 \\ -\sqrt{3}\omega_R(\Gamma - i2(\omega_L - \omega_R)) \\ -\sqrt{3}\omega_R(\Gamma + i2(\omega_L - \omega_R)) \end{pmatrix} \quad \begin{matrix} \rho^{dd} \\ \rho^{cc} \\ \rho^{6} \\ \rho^{dc} \\ \rho^{cd} \end{matrix}$$

with $D = 2\Gamma^2 + 8\omega_L^2 - 12\omega_L\omega_R + 9\omega_R^2$.

$$I = -e4\Gamma\omega_R^2/3D$$

$$F = \frac{16\omega_R^2\left(2\Gamma^2 + 53\omega_L^2\right) - 176\omega_L\omega_R\left(\Gamma^2 + 4\omega_L^2\right)}{3D^2} + \frac{20\left(\Gamma^2 + 4\omega_L^2\right)^2 - 576\omega_L\omega_R^3 + 195\omega_R^4}{3D^2}$$

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Lamb shift at the $5_0 \leftrightarrow 6$ resonance