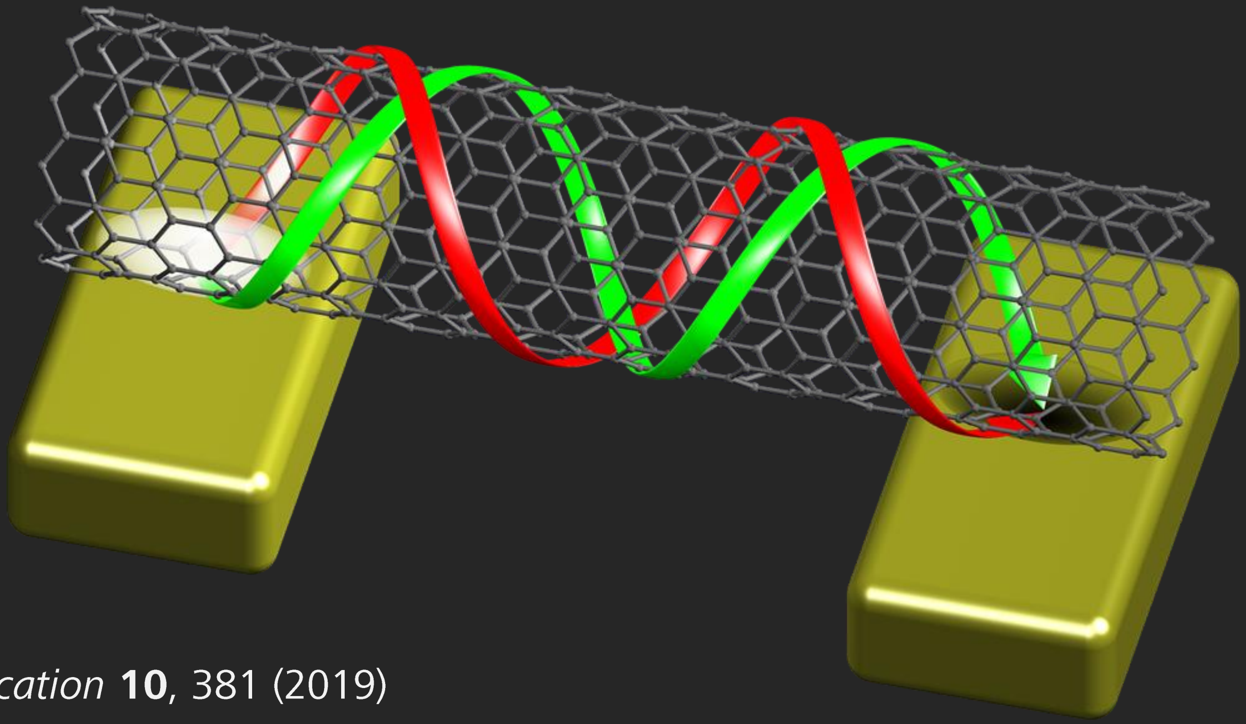
Dark states in a carbon nanotube quantum dot



Nature Communication **10**, 381 (2019)

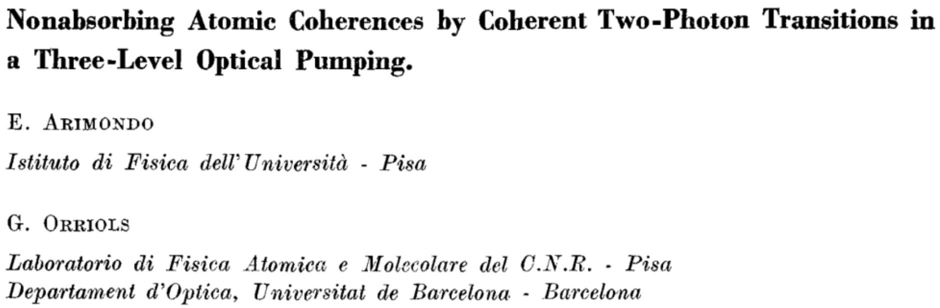
Andrea Donarini

Telluride
30.07.2019

University of Regensburg, Germany

Dark states in optics

G. Alzetta et al. *Nuovo Cimento*. **36**, 5 (1976),
E. Arimondo and G. Orriolis, *Lettere al Nuovo Cimento*, **17**, 33, (1976)



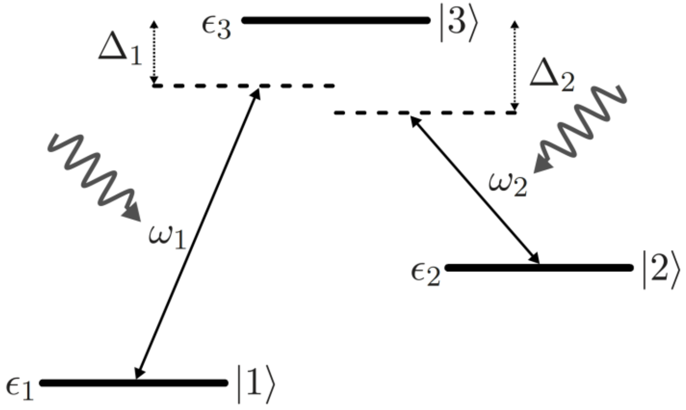
Nonabsorbing Atomic Coherences by Coherent Two-Photon Transitions in a Three-Level Optical Pumping.

E. ARIMONDO

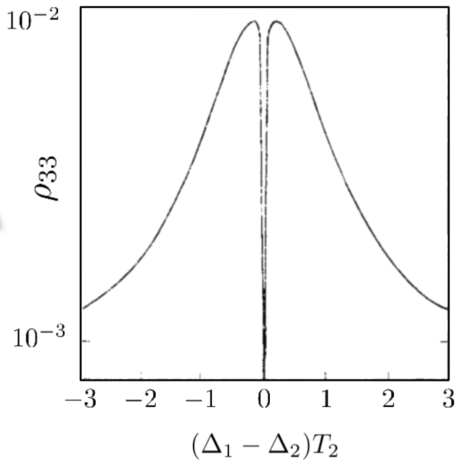
Istituto di Fisica dell'Università - Pisa

G. ORRIOLS

Laboratorio di Fisica Atomica e Molecolare del C.N.R. - Pisa
Departament d'Optica, Universitat de Barcelona - Barcelona

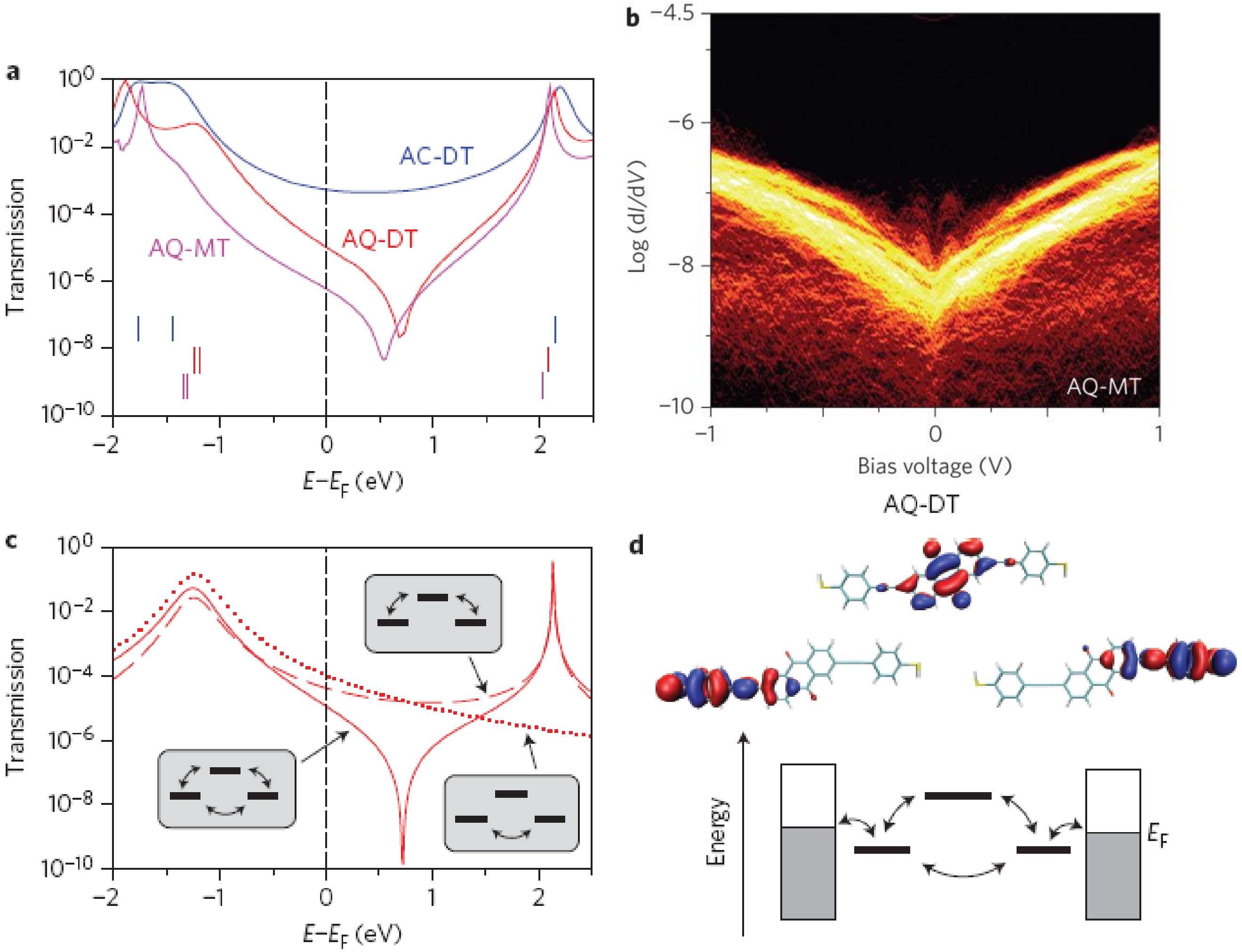


$|\mathrm{DS}\rangle$



Molecular interference

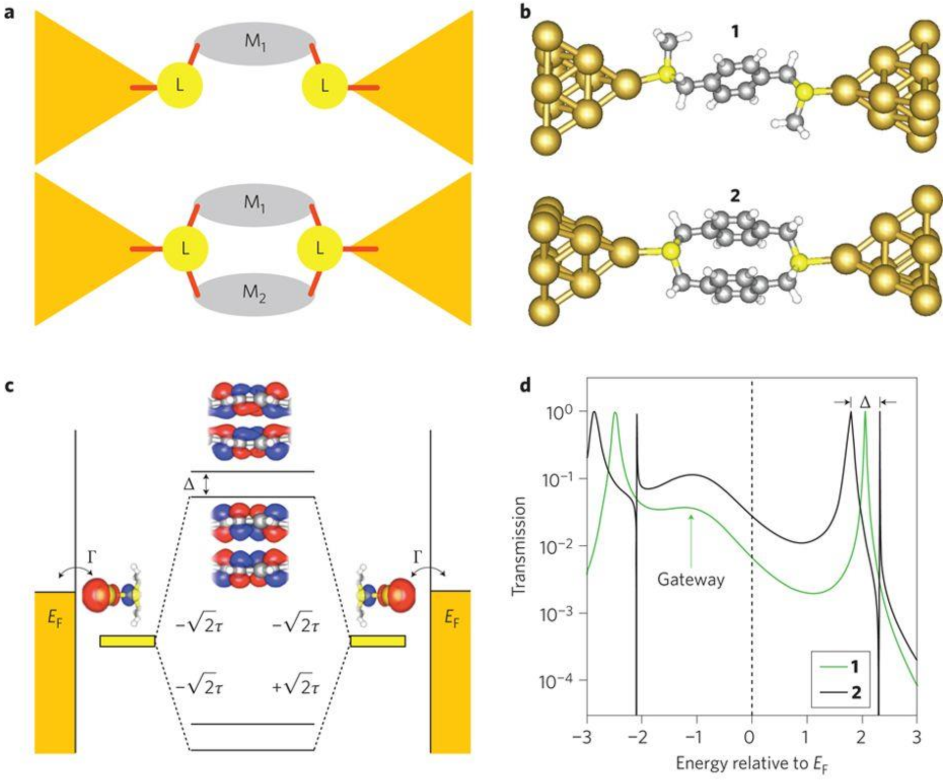
C. M. Guédon, H. Valkenier et al. *Nature Nanotech*. **7**, 305 (2012)

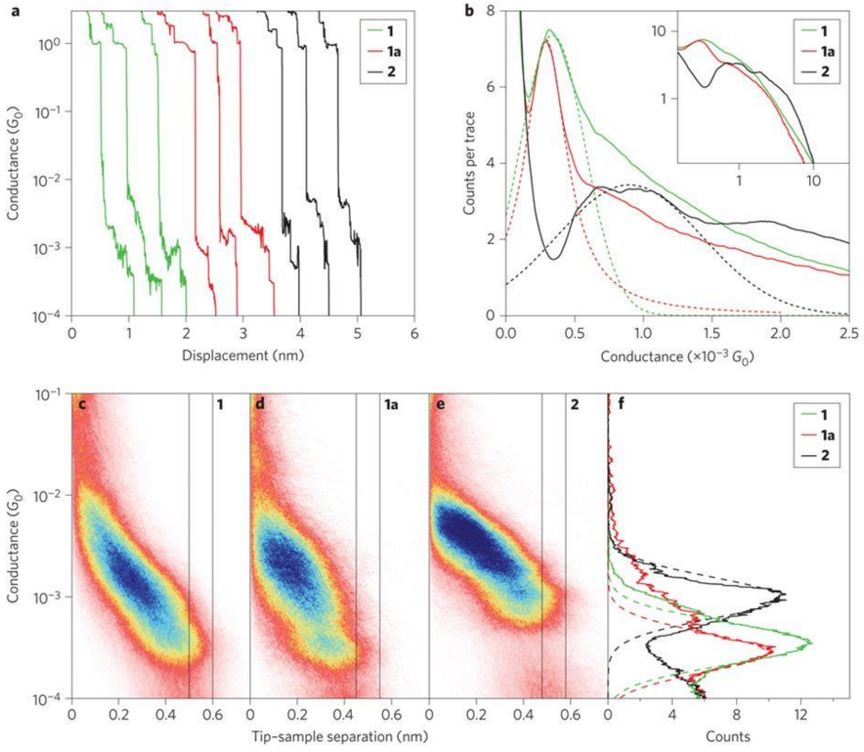


- Zero bias conductance
- Strong coupling to the leads → coherent (single-particle) transport

Parallel paths

H. Vazquez, R. Skouta et al. *Nature Nanotech*. **7**, 663-667 (2012)





PRL **104,** 186804 (2010) PHYSICAL REVIEW LETTERS week ending 7 MAY 2010

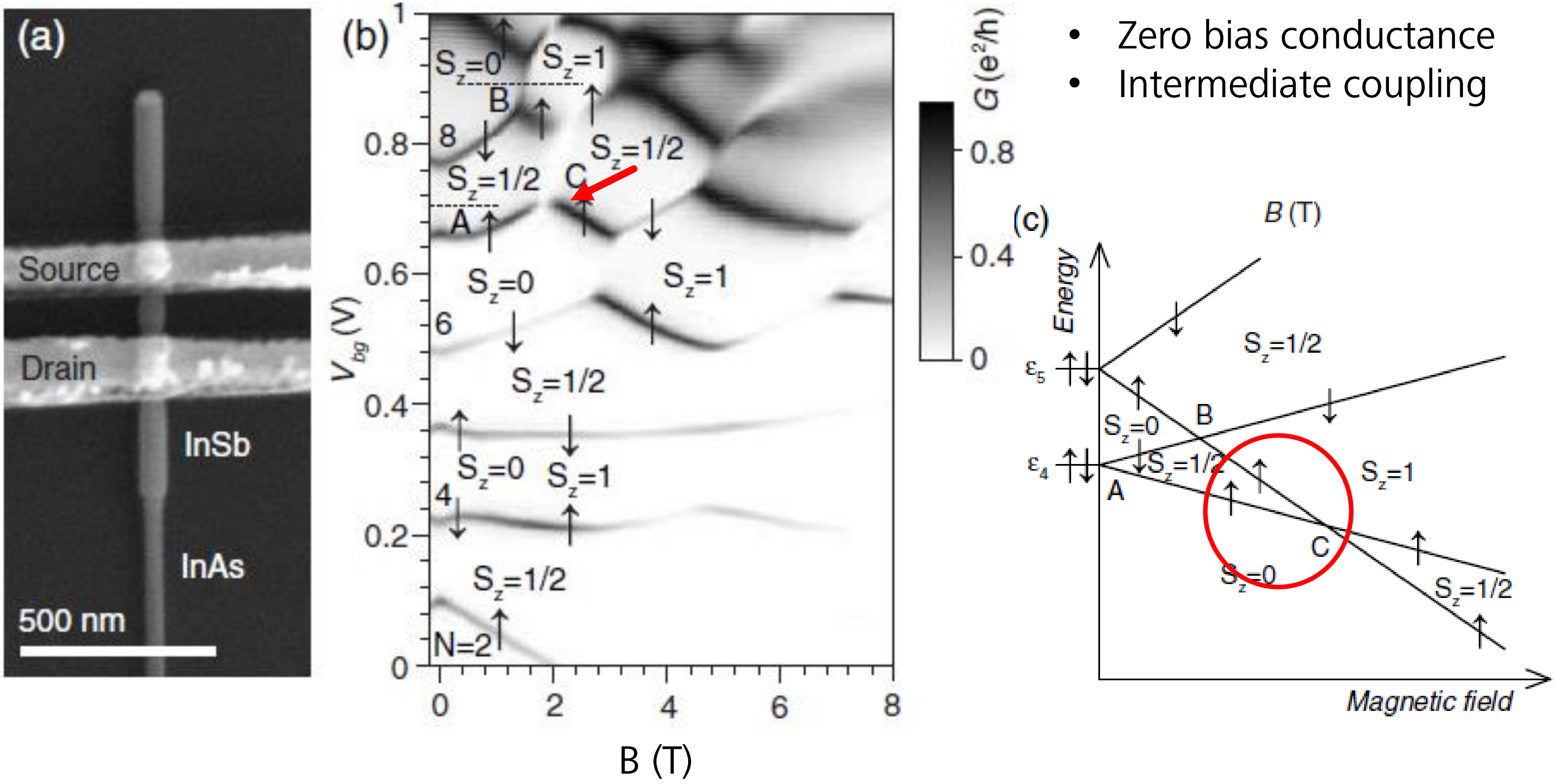
Correlation-Induced Conductance Suppression at Level Degeneracy in a Quantum Dot

H. A. Nilsson, O. Karlström, M. Larsson, P. Caroff, J. N. Pedersen, L. Samuelson, A. Wacker,
L.-E. Wernersson, and H. Q. Xu*

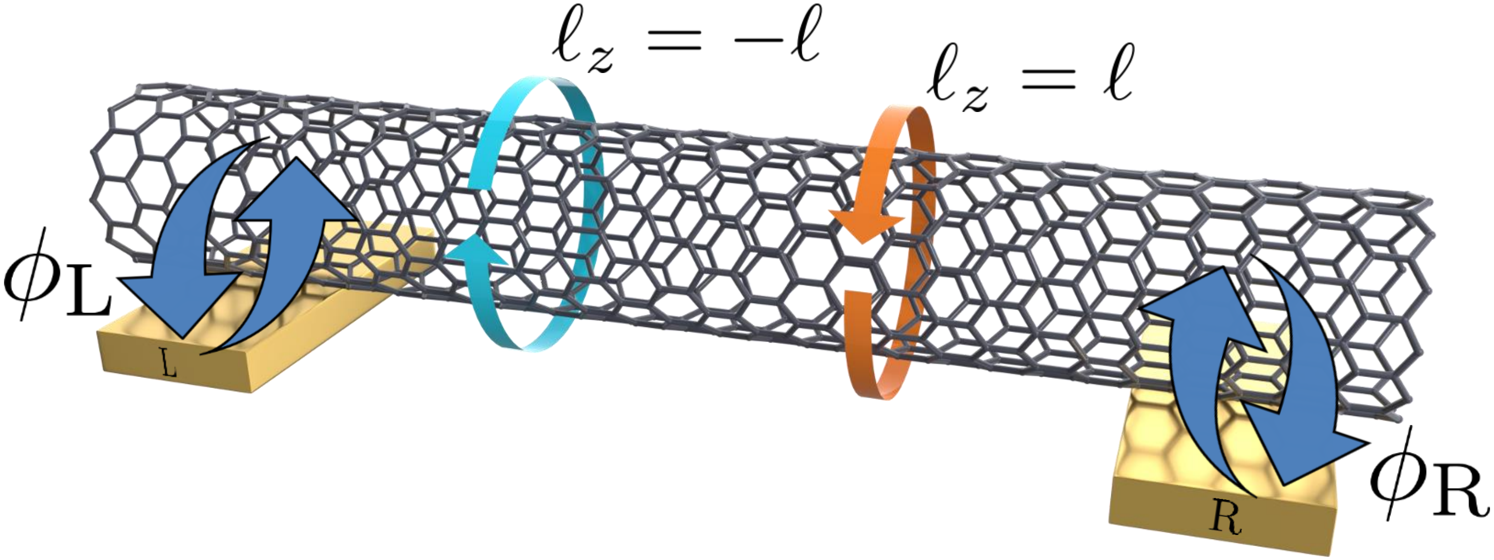
Nanometer Structure Consortium, Lund University, Box 118, 221 00 Lund, Sweden

(Received 10 November 2009; published 4 May 2010)

- Zero bias conductance
- Intermediate coupling

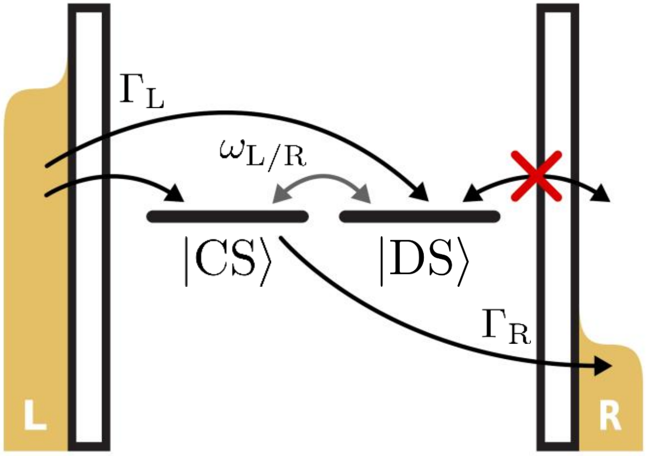


All-electrical dark states



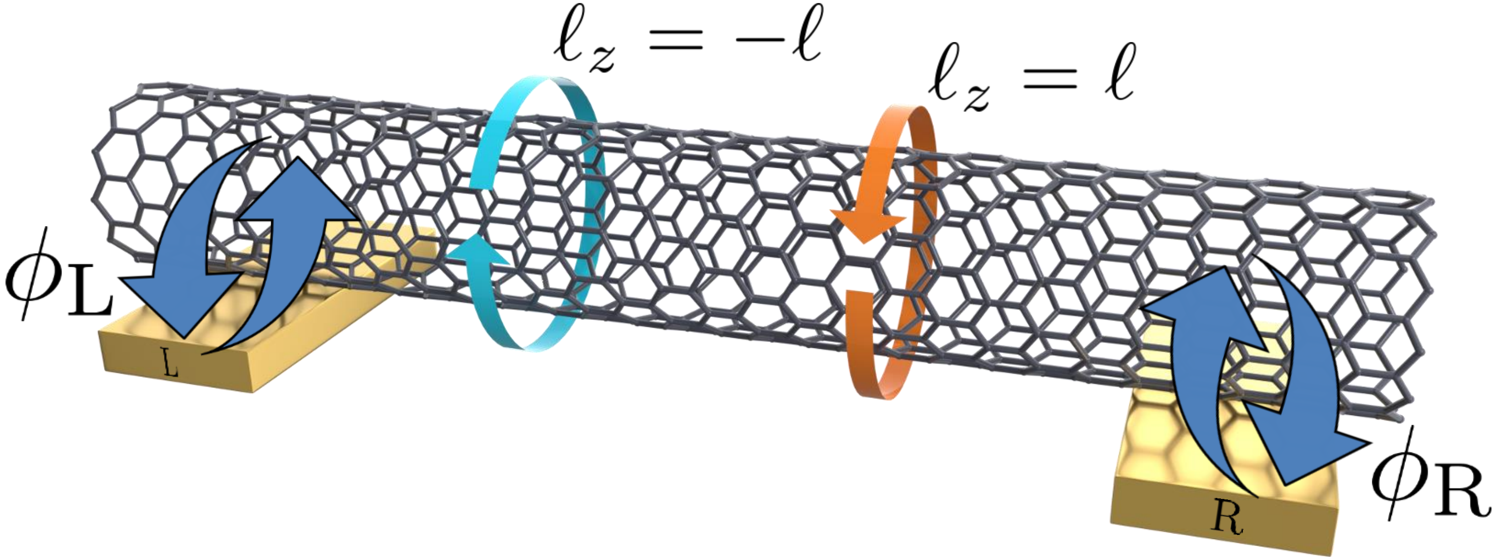
Ingredients:

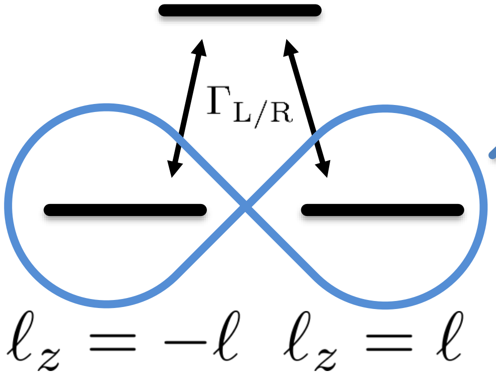
- Orbital (quasi)-degeneracy
- Weak coupling to the leads → charging effects
- Phase coherent tunnelling dynamics
- Finite bias → directed transport



$$\langle 0|d_\mathrm{R}|\mathrm{DS}\rangle = 0$$

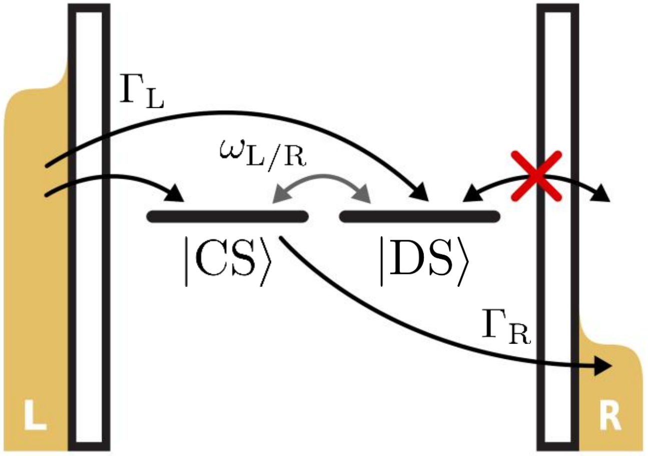
All-electrical dark states





$N = 0$

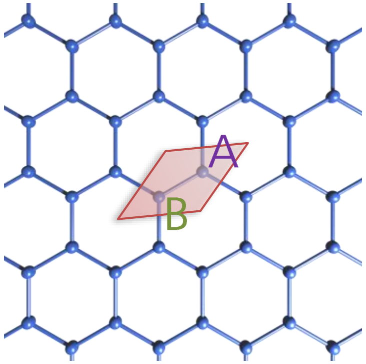
$N = 1 \quad |\mathrm{DS}\rangle$

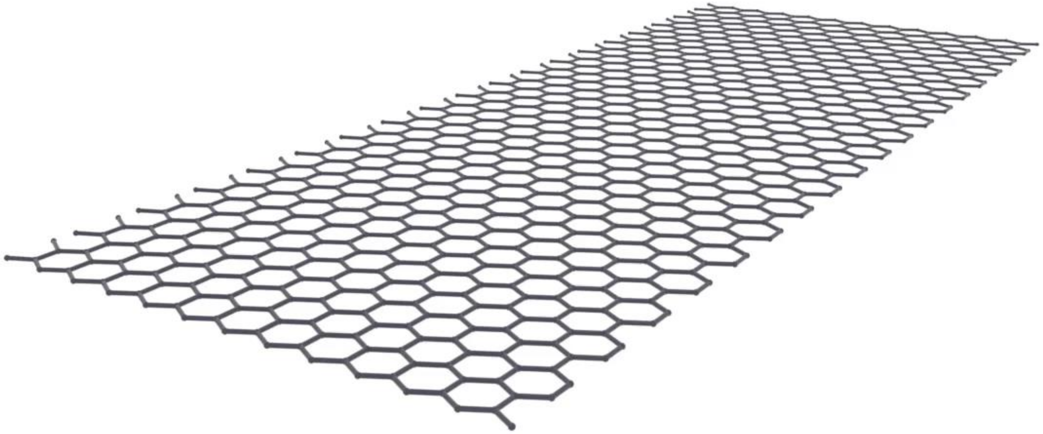


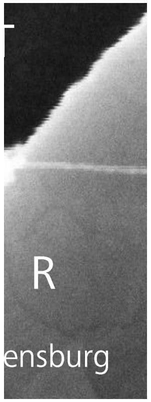
$$\langle 0|d_{\mathrm{R}}|\mathrm{DS}\rangle = 0$$

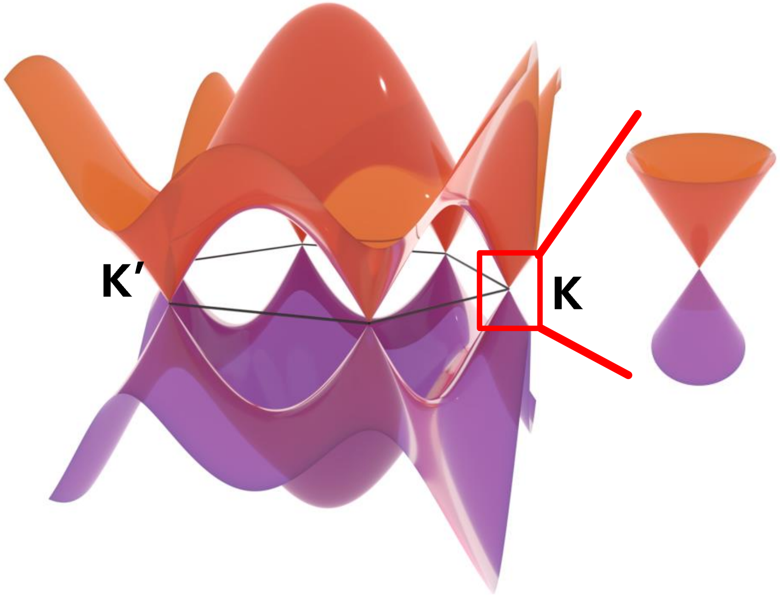
CNT quantum dots

Graphene



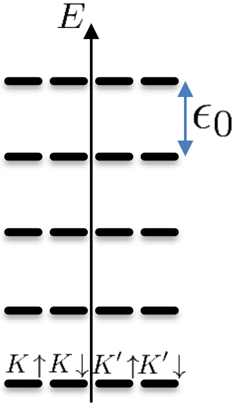






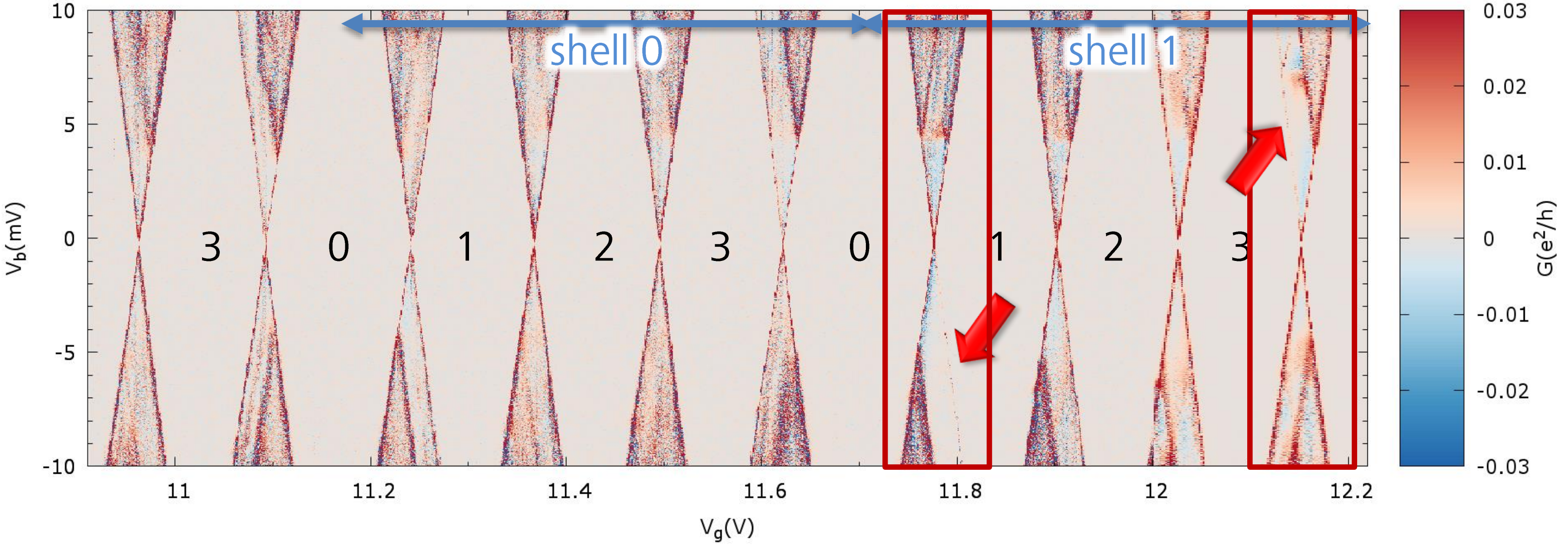
Transverse +
Longitudinal
quantization

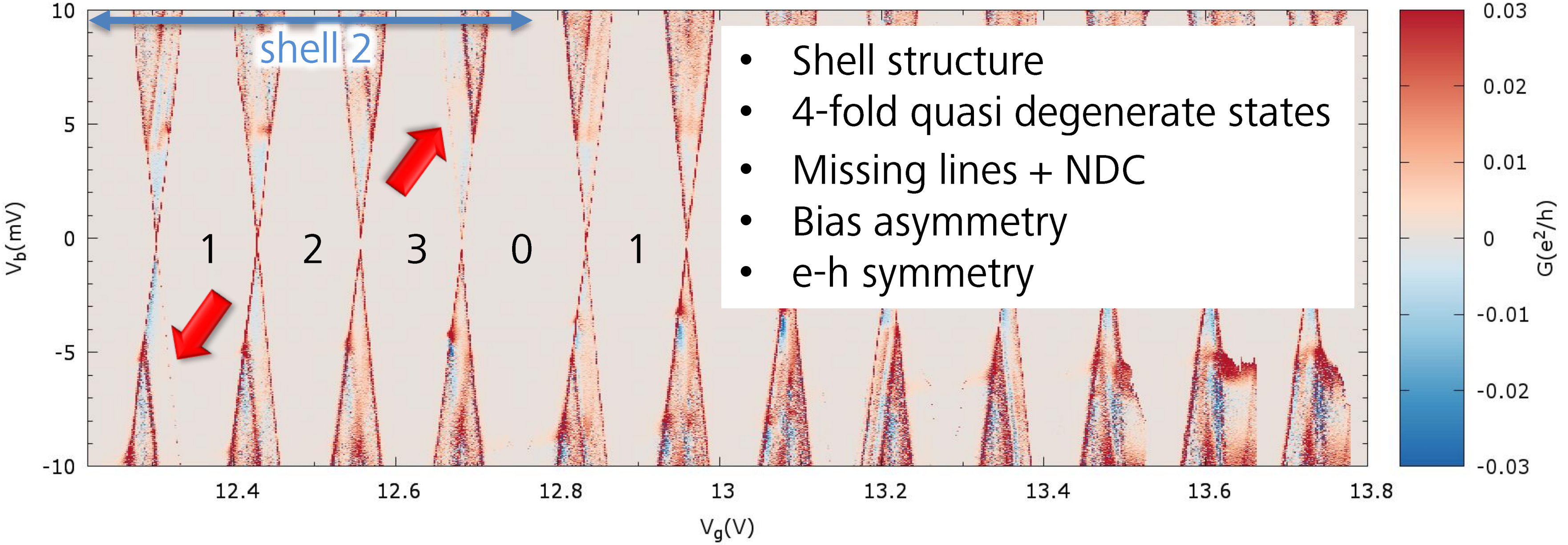
Quantum dot:



- Shell structure
- 4-fold degenerate:
  - 2x spin
  - 2x angular momentum (valley)

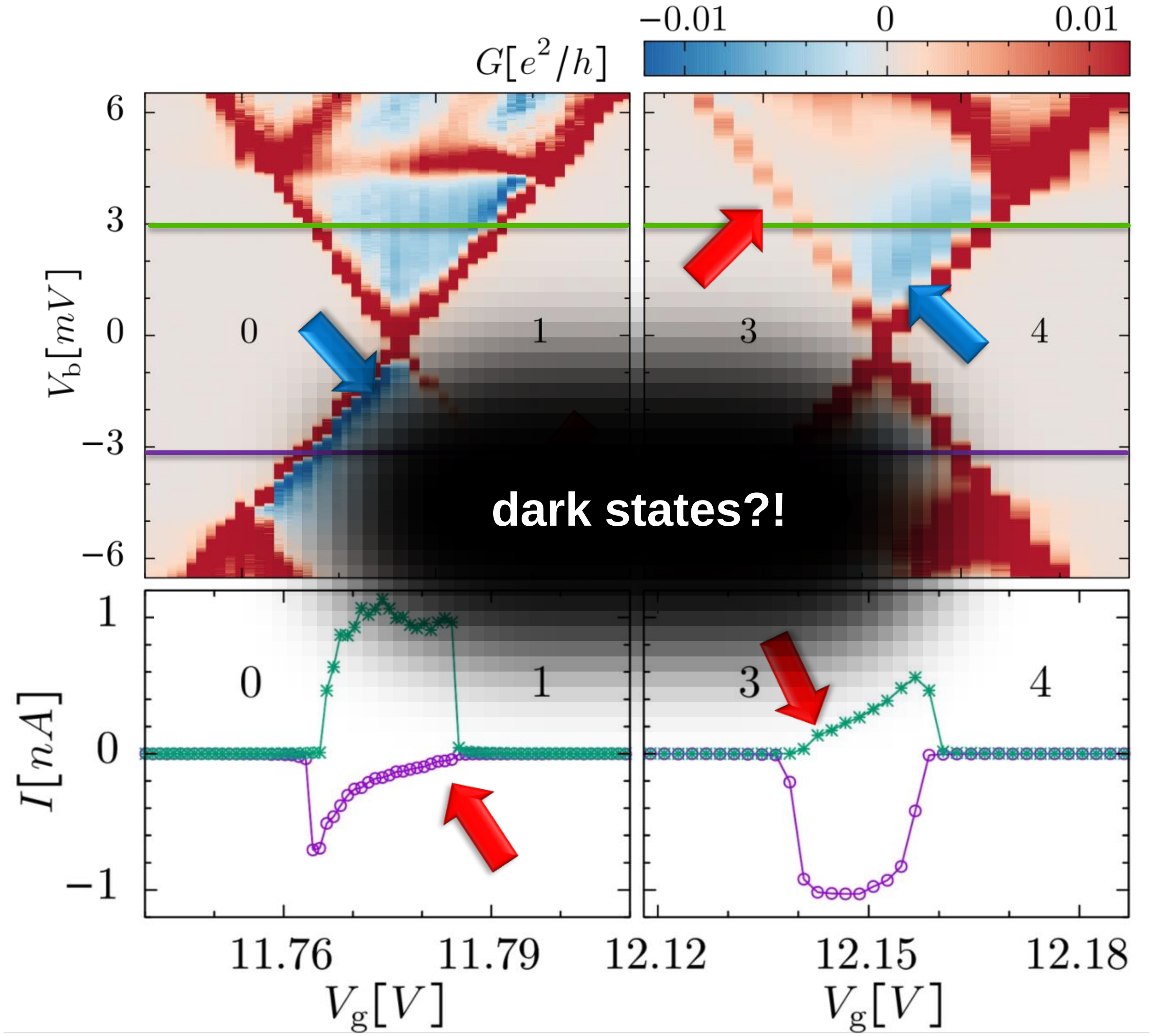
Stability diagram





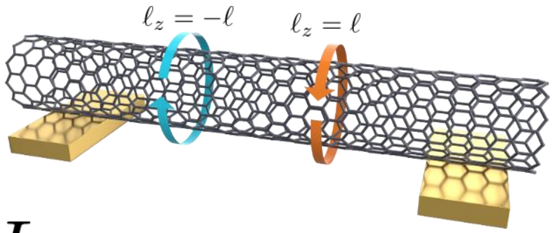
- Shell structure
- 4-fold quasi degenerate states
- Missing lines + NDC
- Bias asymmetry
- e-h symmetry

Dark states in transport



dark states?!

Total Hamiltonian



$$H = H_{\mathrm{CNT}} + H_{\mathrm{leads}} + H_{\mathrm{tun}}$$

$$H_{\mathrm{CNT}} = \sum_{m\ell_z} (m\epsilon_0 - \xi)\, \hat{n}_{m\ell_z} + \frac{U}{2}\hat{N}^2 + J\sum_{m}\left(\hat{\mathbf{S}}_{m\ell}\cdot\hat{\mathbf{S}}_{m-\ell} + \frac{1}{4}\hat{n}_{m\ell}\hat{n}_{m-\ell}\right)$$

Level spacing

V_{g}

Constant interaction

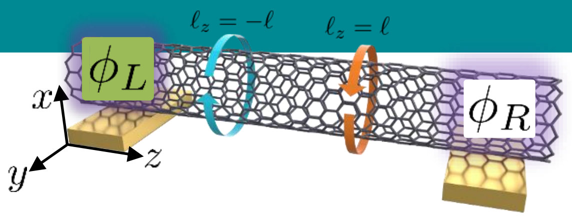
Exchange interaction for zig-zag class CNTs

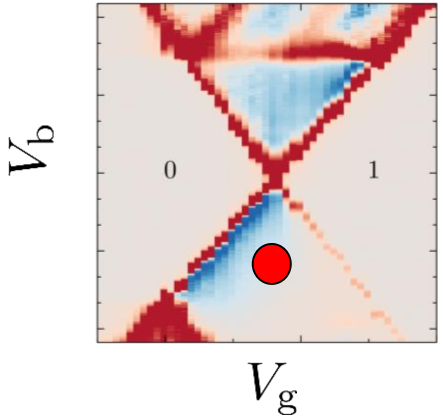
$$H_{\mathrm{leads}} = \sum_{\alpha\mathbf{k}\sigma} \varepsilon_{\mathbf{k}} c^{\dagger}_{\alpha\mathbf{k}\sigma} c_{\alpha\mathbf{k}\sigma}$$

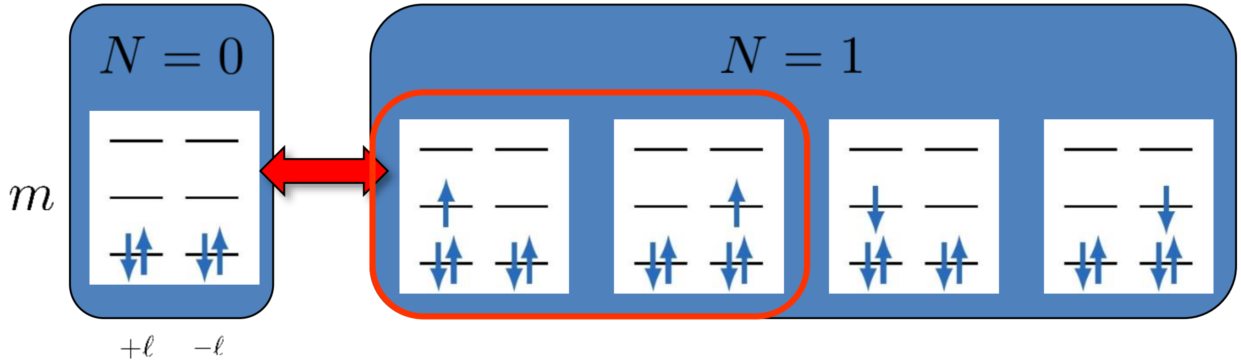
$$\boxed{H_{\mathrm{tun}} = \sum_{\alpha\mathbf{k}m\ell_z\sigma} t_{\alpha\mathbf{k}m\ell_z} d^{\dagger}_{m\ell_z\sigma} c_{\alpha\mathbf{k}\sigma} + \mathrm{h.c.}}$$

Includes the geometry of the contacts.
Is treated perturbatively

One particle dark state







$$\Gamma^{\alpha}_{\ell_z\ell'_z}(E_1-E_0)=\frac{2\pi}{\hbar}\sum_{\mathbf{k}}(t_{\alpha\mathbf{k}\ell_z})^* t_{\alpha\mathbf{k}\ell'_z}\delta(\epsilon_{\mathbf{k}}-E_1-E_0)$$

$$\Downarrow$$

$$\boldsymbol{\Gamma}^{\alpha}=\Gamma^{\alpha}\underbrace{\begin{pmatrix}1 & ae^{2i\ell\phi_\alpha}\\ ae^{-2i\ell\phi_\alpha} & 1\end{pmatrix}}_{\mathcal{R}_l\text{ : coherence matrix}}$$

Atomically localized tunneling
or
Surface Γ-point approximation
(i.e. $k_x \approx k_F \quad k_y,\ k_z = 0$)

$$a = 1$$

Tunneling rate matrix

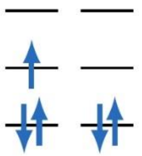
$$\mathcal{R}_\mathrm{R} = \begin{pmatrix} 1 & e^{i2\phi_\mathrm{R}} \\ e^{-i2\phi_\mathrm{R}} & 1 \end{pmatrix} \xrightarrow{\text{diagonalize}} \mathcal{R}_\mathrm{R} = \begin{pmatrix} \enclose{circle}{0} & 0 \\ 0 & 2 \end{pmatrix}$$

$$\mathcal{R}_\mathrm{L} = \begin{pmatrix} 1 & e^{i2\phi_\mathrm{L}} \\ e^{-i2\phi_\mathrm{L}} & 1 \end{pmatrix} \xrightarrow[\Delta\phi = \phi_\mathrm{L} - \phi_\mathrm{R}]{} \mathcal{R}_\mathrm{L} = \begin{pmatrix} 1-\cos\Delta\phi & -i\sin\Delta\phi \\ i\sin\Delta\phi & 1+\cos\Delta\phi \end{pmatrix}$$

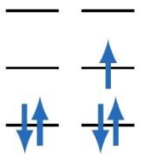
Cannot be diagonalized at the same time if $\Delta\phi \neq n\pi$

The dark state is defined with respect to a **specific** lead

$$|DS,\uparrow\alpha\rangle = \frac{1}{\sqrt{2}}\Big(e^{i\ell\phi_\alpha}$$

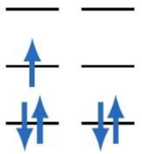


$$- e^{-i\ell\phi_\alpha}$$

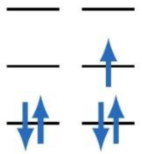


$$\Big)$$

$$|CS,\uparrow\alpha\rangle = \frac{1}{\sqrt{2}}\Big(e^{i\ell\phi_\alpha}$$

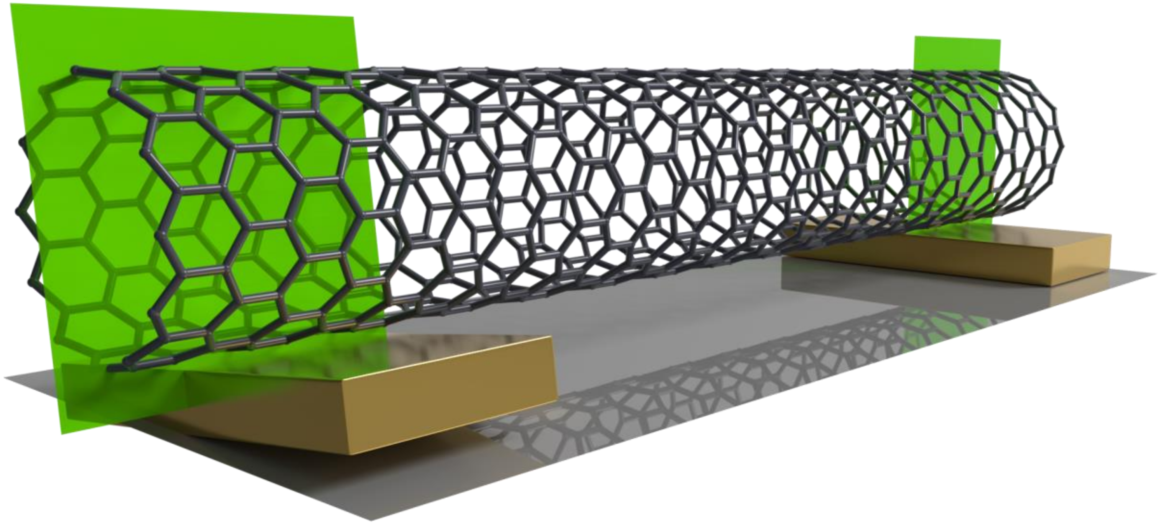


$$+ e^{-i\ell\phi_\alpha}$$



$$\Big)$$

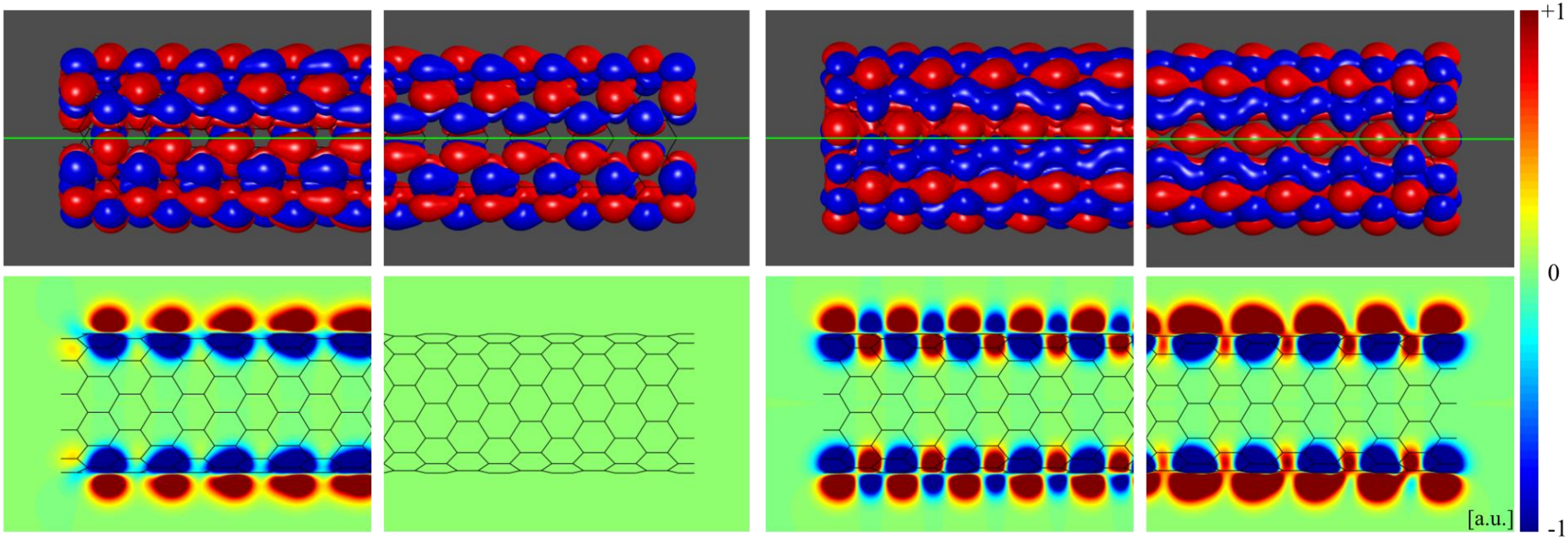
Dark states in CNTs



(12,0) CNT

Dark state

Coupled state



Quantum transport

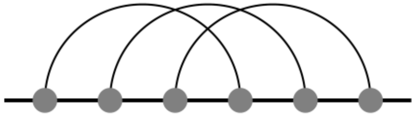
Weak coupling

Increase coupling

Strong coupling

Density matrix approach

$\mathcal{O}(\Gamma) \quad \cdots \quad \mathcal{O}(\Gamma^2) \quad \cdots$



diagrammatics

Current

$\langle \dot{N} \rangle$

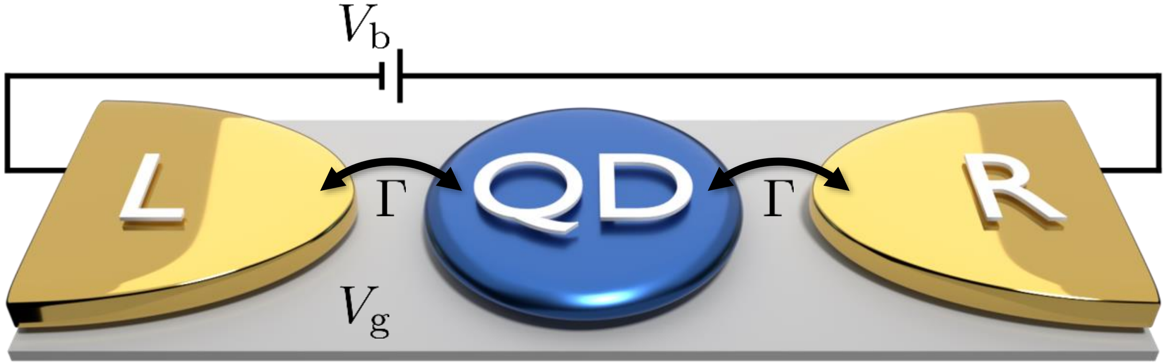
Statistics

Counting field: χ

Full counting statistics

Current cumulants

$$\boxed{\dot{\rho} = \mathcal{L}\rho}$$



Single electron transistor

Quantum transport

Master equation:

$$0 = \mathcal{L}\rho^{\infty} = -\frac{i}{\hbar}\left[\hat{H}_{\mathrm{CNT}} + \hat{H}_{\mathrm{LS}}, \rho^{\infty}\right] + \mathcal{L}_{\mathrm{tun}}\rho^{\infty} + \mathcal{L}_{\mathrm{rel}}\rho^{\infty}$$

Reduced density matrix

Lamb shift

tunneling

relaxation

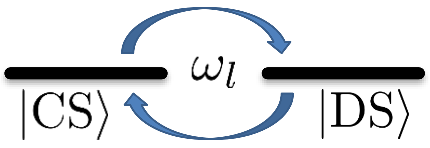
Minimal model for $N = 0 \leftrightarrow 1$ (no extrinsic relaxation):

$$0 = \dot{\rho}_1 = -\frac{i}{\hbar}\left[\hat{H}_{\mathrm{LS}}, \rho_1\right] + 2\Gamma_{\mathrm{L}}\mathcal{R}_{\mathrm{L}}\rho_0 - \frac{\Gamma_{\mathrm{R}}}{2}\left\{\mathcal{R}_{\mathrm{R}}, \rho_1\right\}$$

$$0 = \dot{\rho}_0 = \Gamma_{\mathrm{R}}\mathrm{tr}\left\{\mathcal{R}_{\mathrm{R}}\rho_1\right\} - 4\Gamma_{\mathrm{L}}\rho_0$$

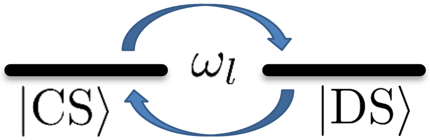
Lamb shift:

$$\hat{H}_{\mathrm{LS}} = \frac{\hbar}{2}\sum_{l}\omega_l\mathcal{R}_l$$



Lamb shift

$$\hat{H}_{\rm LS} = \frac{\hbar}{2}\sum_l \omega_l \mathcal{R}_l$$



$$\omega_l(V_{\rm g}, V_{\rm b}) = \frac{\Gamma_l}{\pi}\left[p_l(\alpha_{\rm g}V_{\rm g}) - p_l(U - \frac{J}{2} + \alpha_{\rm g}V_{\rm g})\right]$$

$$p_l(\Delta E) = -{\rm Re}\psi\left(\frac{1}{2} + i\frac{\Delta E - \mu_l}{2\pi k_{\rm B}T}\right)$$

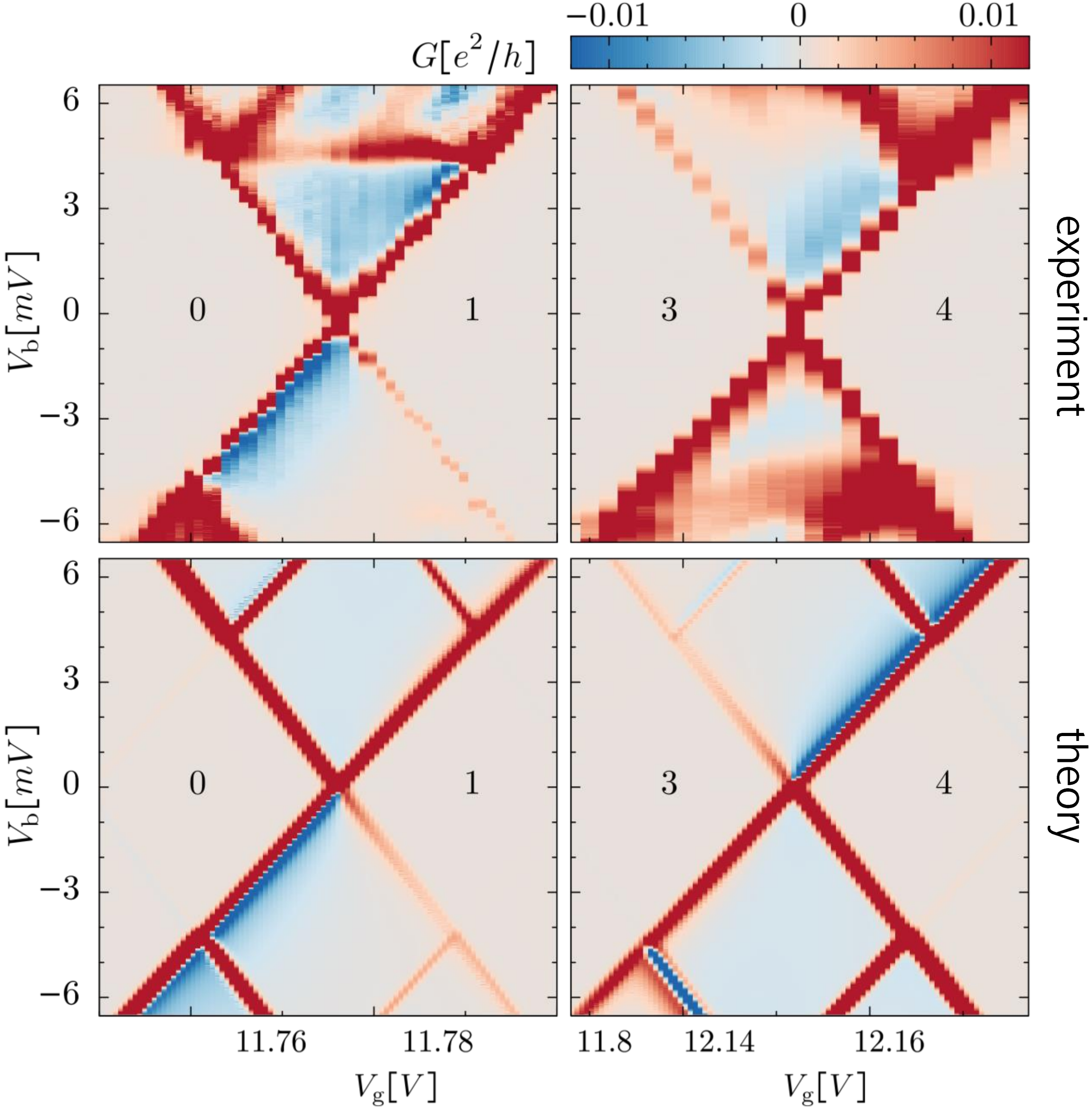
$$\mu_{\rm L} = \eta e V_{\rm b}$$
$$\mu_{\rm R} = (\eta - 1)eV_{\rm b}$$

Digamma function

Terms come from principal value integrals over the Fermi functions:

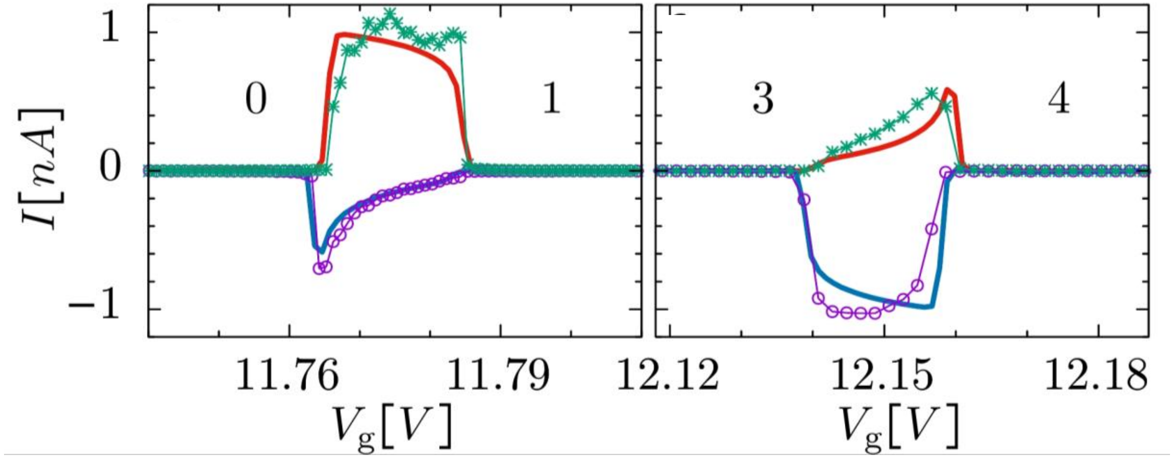
$$p.v. \int_{-\infty}^{\infty} {\rm d}\epsilon \frac{f^{\pm}(\epsilon)}{\epsilon - \Delta E}$$

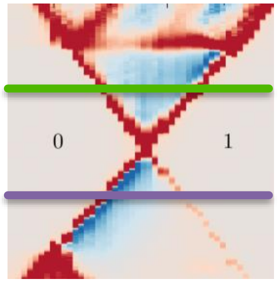
Compare to experiment



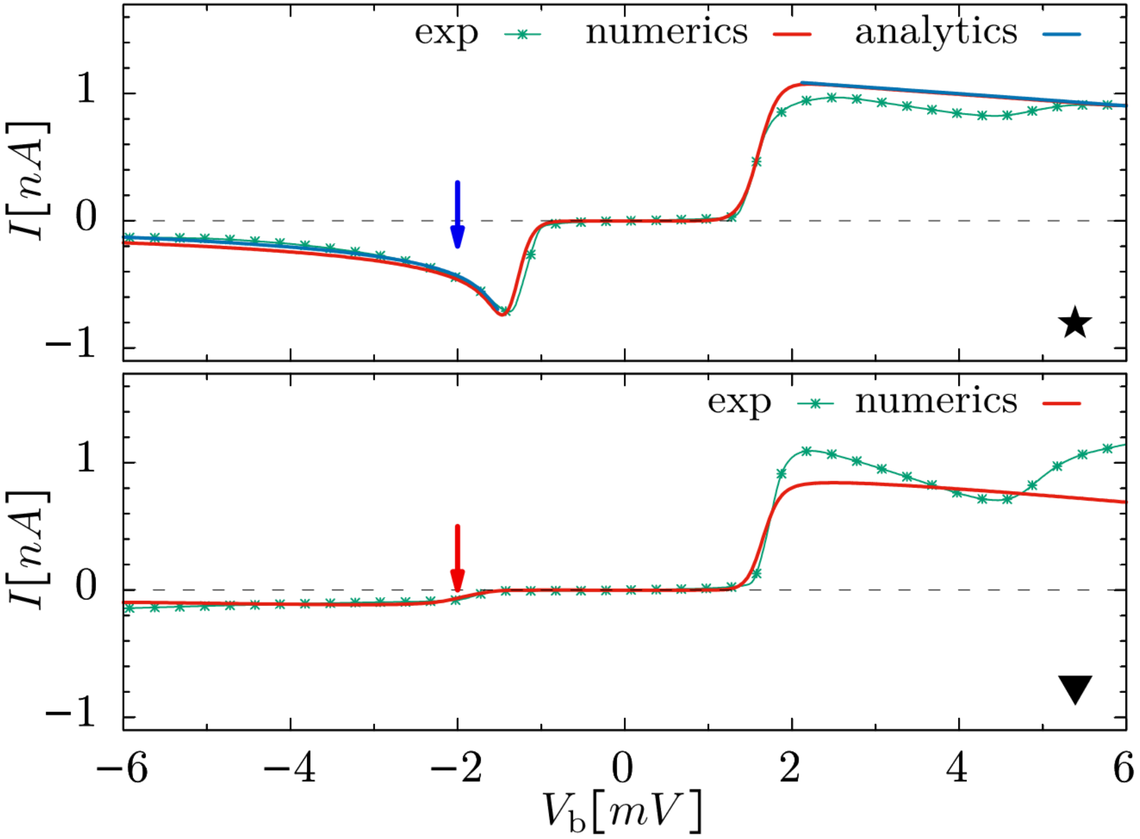
Compare to experiment

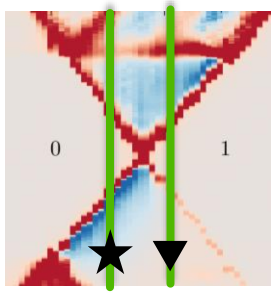
Gate traces:



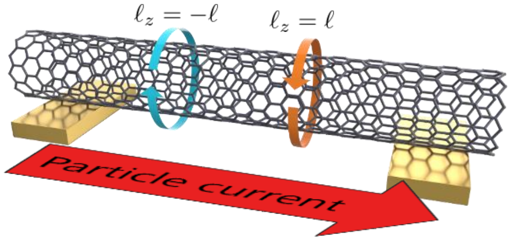


Bias traces:





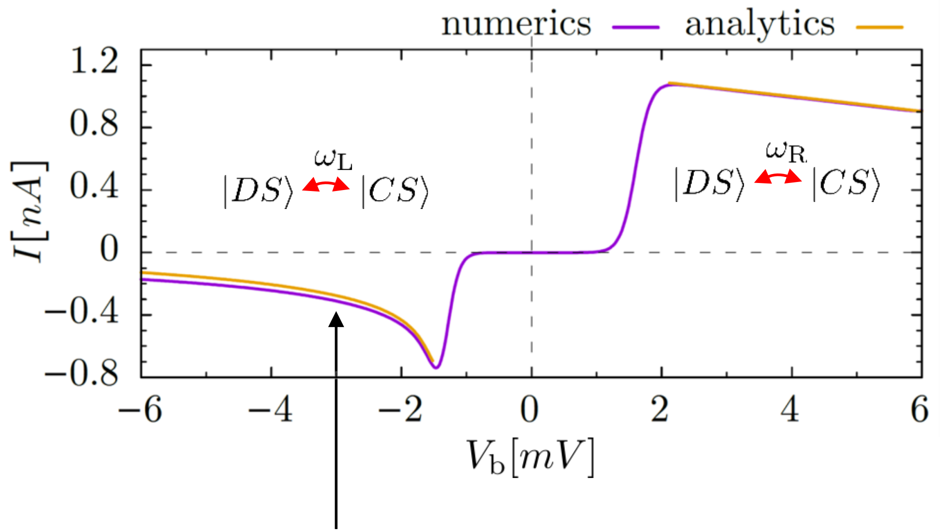
Lamb-shift precession

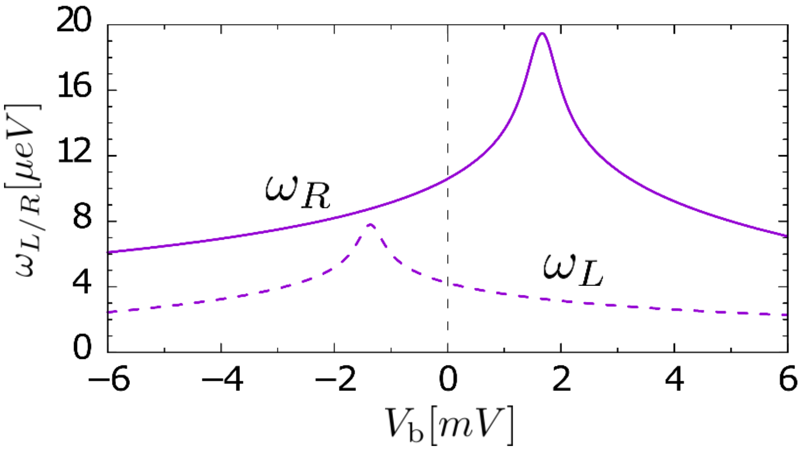


$$\dot{\rho} = -\frac{i}{\hbar}[H_{\mathrm{CNT}}, \rho] - \frac{i}{\hbar}[H_{\mathrm{LS}}, \rho] + \mathcal{L}_{\mathrm{tun}}[\rho] + \mathcal{L}_{\mathrm{rel}}[\rho]$$

$$H_{\mathrm{LS}} = \frac{\hbar}{2}(\omega_L \mathcal{R}_L + \omega_R \mathcal{R}_R)$$

$$\Big[\mathcal{R}_\alpha, |DS,\alpha\rangle\langle DS,\alpha|\Big] = 0$$ ➡ Only the **source** contribution perturbs the dark state

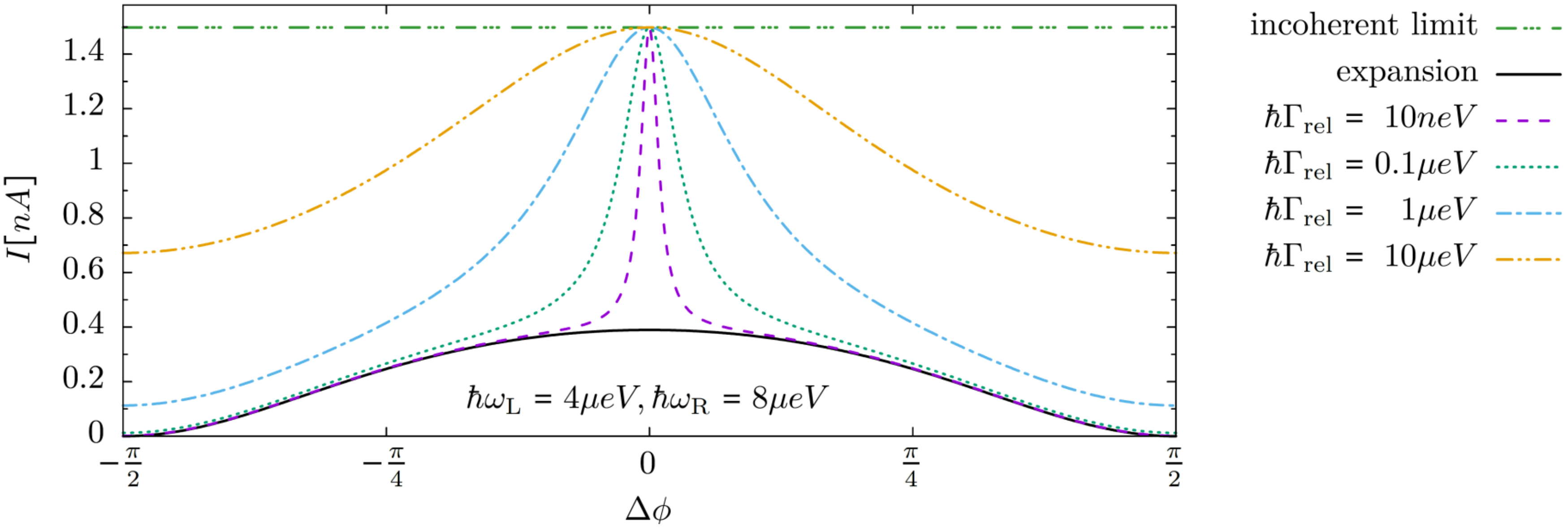




$$I = \frac{4e\Gamma_{\mathrm{R}}\omega_{\mathrm{L}}^2(1+\cos\Delta\phi)}{\Gamma_{\mathrm{R}}^2 + 4(\omega_{\mathrm{L}} - \omega_{\mathrm{R}})^2 + \omega_{\mathrm{L}}(1+\cos\Delta\phi)\left[\omega_{\mathrm{L}}\Gamma_{\mathrm{R}}/\Gamma_{\mathrm{L}} + 4\omega_{\mathrm{R}}\right]} \qquad \Delta\phi = \phi_L - \phi_R$$

Current vs. tunneling phase

$$I = \frac{4e\Gamma_{\mathrm{R}}\omega_{\mathrm{L}}^2(1+\cos\Delta\phi)}{\Gamma_{\mathrm{R}}^2 + 4(\omega_{\mathrm{L}}-\omega_{\mathrm{R}})^2 + \omega_{\mathrm{L}}(1+\cos\Delta\phi)\left[\omega_{\mathrm{L}}\Gamma_{\mathrm{R}}/\Gamma_{\mathrm{L}} + 4\omega_{\mathrm{R}}\right]} \qquad \Delta\phi = \phi_L - \phi_R$$



$$I(\Delta\phi = 0) = \frac{4e\Gamma_{\mathrm{L}}\Gamma_{\mathrm{R}}}{4\Gamma_{\mathrm{L}} + \Gamma_{\mathrm{R}}}$$

incoherent current through 4 degenerate levels

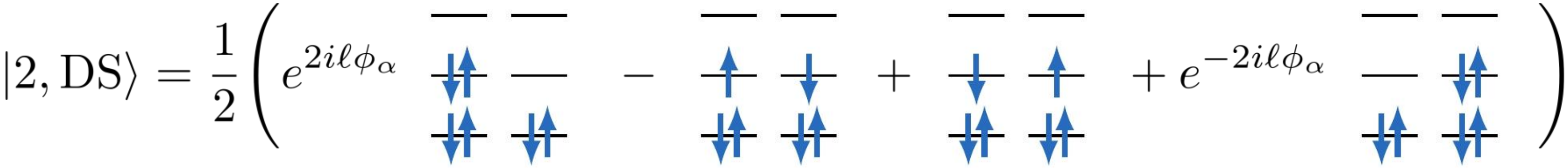
Exchange interaction

$$H_{\mathrm{CNT}} = \sum_{m\ell_z} (m\epsilon_0 - \xi)\,\hat{n}_{m\ell_z} + \frac{U}{2}\hat{N}^2 + J\sum_{m}\left(\hat{\mathbf{S}}_{m\ell}\cdot\hat{\mathbf{S}}_{m-\ell} + \frac{1}{4}\hat{n}_{m\ell}\hat{n}_{m-\ell}\right)$$

$J = 0$

2-particle groundstate is 6-fold degenerate

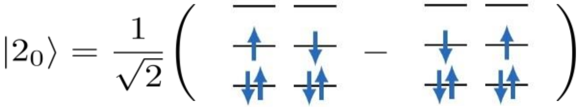
➡ new dark state possible



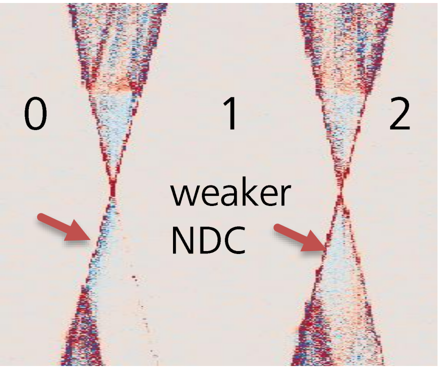
➡ interference NDC also at 1↔2 particle resonance

$J > \Gamma$

2-particle groundstate is the inter-valley singlet



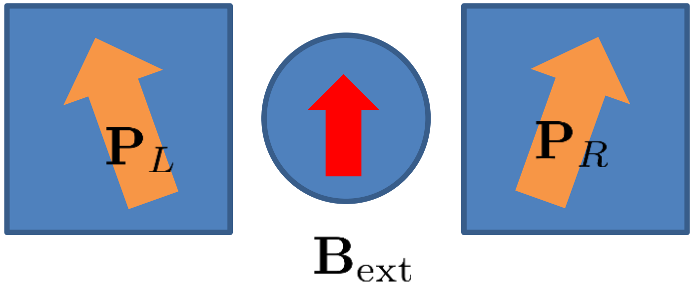
➡ no 2 particle dark state is possible

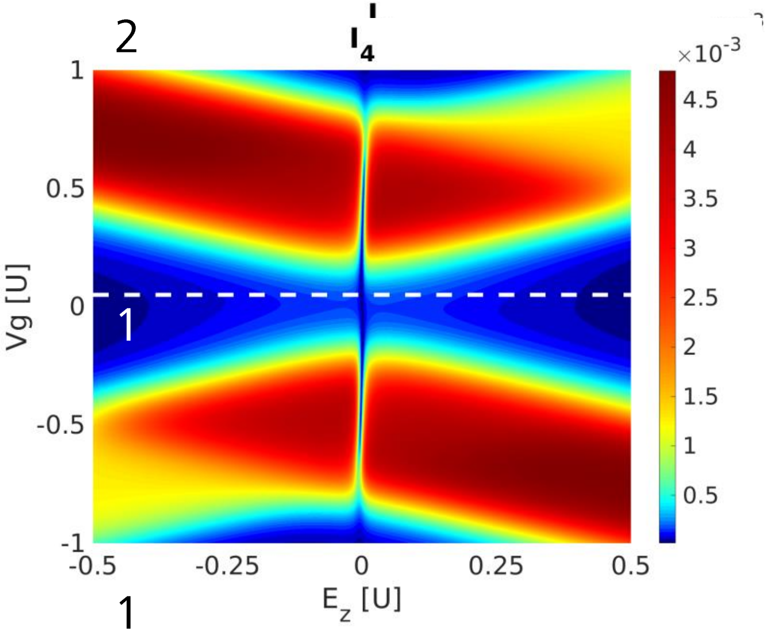


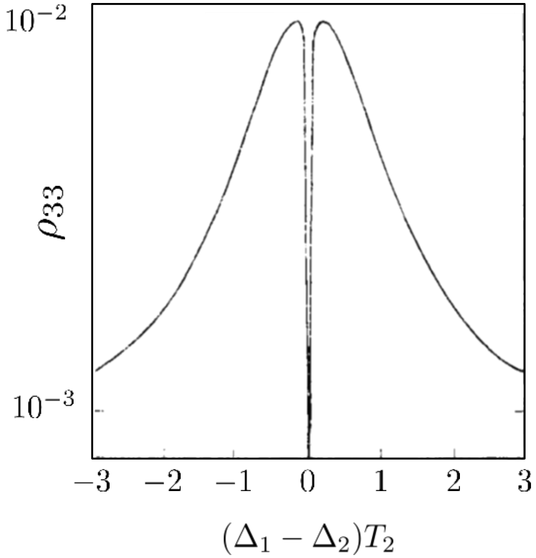
All-electrical dark state

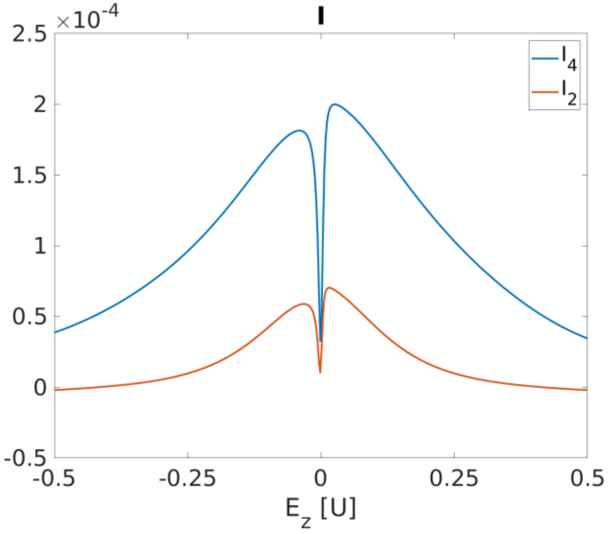
The general tunnelling matrix for a degenerate two level system

$$\boldsymbol{\Gamma}^{\alpha} = \Gamma^{\alpha}(\mathbb{1} + \mathbf{P}_{\alpha} \cdot \boldsymbol{\sigma})$$









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Conclusions

Interference occurs between the **degenerate** angular momentum states of a **zig-zag class** carbon nanotube

Spatially confined tunneling mixes angular momentum states providing a non-diagonal **tunnelling rate matrix**

The **dark state** is the eigenvector of the tunneling rate matrix with zero eigenvalue

The **Lamb-shift** precession perturbs the dark state and explains the **bias voltage asymmetry**

Thank you for your attention