Quantum Theory of Condensed Matter

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Room H33
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Sheet 11

1. Stoner model of metallic ferromagnets

The Stoner model is applied to those materials for which the magnetism is generated by the conduction electrons. They are typically transition metals in which the conduction band is formed by the narrower d or f orbitals. The metallicity of the system allows to introduce a certain degree of screening. In the low momentum regime it is thus reasonable to introduce for this systems a (continuous) Hubbard Hamiltonian.

1. Consider the effective Hamiltonian for a system of interacting electrons written in first quantization:

$$H = -\sum_i \frac{\nabla_i^2}{2m} + \sum_{ij} U\delta(\mathbf{r}_i - \mathbf{r}_j)$$

and write it in second quantization in the position and in the momentum basis. The former will closely resemble the Hubbard Hamiltonian already encountered during the lectures.

2. Apply the Hartree-Fock approximation on this Hamiltonian keeping in mind that we are looking for ferromagnetic solutions. That is parametrize the spin up and spin down populations:

$$\langle c^\dagger_{\mathbf{k}\uparrow} c_{\mathbf{k}'\uparrow}\rangle = \delta_{\mathbf{k}\mathbf{k}'}\bar{n}_{\mathbf{k}\uparrow}, \quad \langle c^\dagger_{\mathbf{k}\downarrow} c_{\mathbf{k}'\downarrow}\rangle = \delta_{\mathbf{k}\mathbf{k}'}\bar{n}_{\mathbf{k}\downarrow}$$

and assume that the average populations $\bar{n}_{\mathbf{k}\uparrow}$ and $\bar{n}_{\mathbf{k}\downarrow}$ of spin up and down electrons respectively can have different values.

3. Write the the self-consistency conditions:

$$\bar{n}_\sigma = \frac{1}{V}\sum_{\mathbf{k}} \langle c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma}\rangle_{\mathrm{MF}}$$

for the spin up and down respectively. V is the volume of the crystal. Hint: At zero temperature you should obtain:

$$\bar{n}_\uparrow = \int \frac{\mathrm{d}\mathbf{k}}{(2\pi)^3}\,\theta\left(\mu - \frac{\hbar^2 k^2}{2m} - U\bar{n}_\downarrow\right)$$

for one spin component and similarly for the other. Here θ is the Heaviside function ($\theta(x \geq 0) = 1$, $\theta(x < 0) = 0$). Extend the result to finite temperatures.

4. The average spin up and down densities are connected by the self-consistency conditions just derived. Assume for the moment the $T = 0$ condition and write explicitly the system of coupled equation in $\bar{n}_\uparrow$ and $\bar{n}_\downarrow$ represented by the self-consistency equations. Hint: It could be useful to introduce spin resolved Fermi momenta defined as:

$$\frac{\hbar^2}{2m}k^2_{\mathrm{F}\uparrow} + U\bar{n}_\downarrow = \mu$$
$$\frac{\hbar^2}{2m}k^2_{\mathrm{F}\downarrow} + U\bar{n}_\uparrow = \mu$$

5. Rewrite the self-consistent problem in terms of the variables:

$$\zeta = \frac{\bar{n}_\uparrow - \bar{n}_\downarrow}{\bar{n}_\uparrow + \bar{n}_\downarrow}$$
$$\gamma = \frac{8\pi^{2/3} m U (\bar{n}_\uparrow + \bar{n}_\downarrow)^{1/3}}{3^{2/3}\hbar^2}$$

The physical meaning of ζ is to quantify the excess magnetization since $-1 \leq \zeta \leq 1$. We can call the system *ferromagnetic* when $|\zeta| = 1$ and paramagnetic when $\zeta = 0$. For which values of γ are these special cases ($|\zeta| = 0,\, 1$) obtained? Can you give a physical interpretation of the result?

Frohes Schaffen!