University of Regensburg Summer Term 2010

Quantum Theory of Condensed Matter I

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Sheet 8

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1. Simplified Anderson Impurity Model (AIM)

Let us consider a quantum dot described by a single interacting level, namely the Hamiltonian:

$$H = \sum_\sigma \varepsilon_0 c^\dagger_\sigma c_\sigma + U n_\uparrow n_\downarrow$$

1. Write the retarded single particle Green's function for the system and derive the equation of motion for it. (2 Points)
2. In the previous point you have generated a two particle retarded Green's function: do you recognize it? Now write the equation of motion also for this two particle Green's function. It is useful at this point to transform the system of differential equations from the time to the (Fourier conjugated) frequency domain. The system of equations is now algebraic. Calculate the formal solution of the system. The single particle Green's function of the system still depends on a "parameter": which one? (2 Points)
3. Starting from the solution calculated at the previous point deduce the form of the spectral function. (2 Points)
4. Given that the Hamiltonian is invariant for spin rotation demonstrate that the following relation holds: $\langle n_\uparrow \rangle = \langle n_\downarrow \rangle$. (2 Points)
5. Calculate the form of $\langle n \rangle \equiv \langle n_\uparrow \rangle = \langle n_\downarrow \rangle$ as a function of the on-site energy ε_0 making use of the relation that connects the average density to the spectral function:

   $$\langle n_\sigma \rangle = \int_{-\infty}^{+\infty} \frac{\mathrm{d}\omega}{2\pi} A_\sigma(\omega; \varepsilon_0, \langle n_{\bar\sigma} \rangle) n_F(\omega)$$

   where $n_F(\omega)$ is the Fermi-Dirac distribution function. You can assume for simplicity the chemical potential to be 0 without loss of generality.

   (2 Points)
6. Make a plot of the average number of electron as a function of the on site energy ε_0 and of the spectral function $A_\sigma(\omega; \varepsilon_0)$ as a function of ω for different values of ε_0. One can distinguish 3 different intervals for ε_0 corresponding to three different physical situation. Identify the three intervals and comment on the three regimes.

   (2 Points)

2. Mean field approximation of the simplified AIM

Let's consider the same model of the previous exercise but this time solve it in mean field.

1. Write again the equation of motion for the retarded single particle Green's function and factorize the two-particle Green's function as follows:

   $$\langle\langle n_{\bar\sigma} c_\sigma, c^\dagger_\sigma \rangle\rangle \approx \langle n_{\bar\sigma} \rangle \langle\langle c_\sigma, c^\dagger_\sigma \rangle\rangle$$

where

$$\langle\langle A, B\rangle\rangle \equiv -i\theta(t)\int_{-\infty}^{+\infty} \mathrm{d}t\langle\{A(t), B\}\rangle e^{i\omega t}$$

What is the single particle Green's function in this limit? And the spectral function?

(2 Points)

2. Using the same relation between average density and Green's function as the one introduced in exercise 1 derive the "self-consistency" equations for the two spin average densities on the dot.(N.B. Disregard for the moment the exact relation between the densities for the different spin species.)

(2 Points)

3. Solve, in the limit of zero temperature, the equation derived at the previous point. Make a plot of the different densities as a function of the on-site energy ε_0.

(2 Points)

Frohes Schaffen!