University of Regensburg Summer Term 2016

Applications of Group Theory

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Lectures	H33, Mondays, 14:15
	H34, Thursdays, 14:15
Exercises	5.0.21, Wednesdays, 13:15

Sheet 2

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1. Group of the Hamiltonian

Consider the linear hermitian operator $\hat{H}$ (in practice the Hamiltonian!) that maps a given Hilbert space $\mathcal{H}$ into itself. Prove that the set of all linear, regular operators $\hat{R}$ defined on the same Hilbert space and with the property $[\hat{R}, \hat{H}] = 0$ form a group. Take as binary composition the usual multiplication between operators.

2. Conservation of the norm

Consider a vectorial space V on which a scalar product is defined as a bilinear function by the relation $<e_i, e_j>= \delta_{ij}$, where δ_{ij} is the Kronecker function, and $i(j) = 1, \ldots, n$ labels the n elements $e_{i(j)}$ of a complete basis for V. Prove that each linear transformation f in V which conserves the scalar product between vectors, *i.e.* $<f(v), f(w)>=<v, w>$ is represented by a unitary matrix *i.e.* $M_f M_f^\dagger = M_f^\dagger M_f = \mathbf{1}$, where $\mathbf{1}$ represents the identity matrix.

3. Matrix representations

In the lecture we have introduced the homomorphism connecting point groups to groups of 3x3 matrices generating linear mappings of $\mathbb{R}^3$ into itself. Moreover we related the latter to a group of functionals which can eventually be mapped back into a matrix group once a vectorial space of functions left invariant under the group of functionals is introduced. Let us now make a concrete example:

1. Construct the matrix that generates, in $\mathbb{R}^3$, C_4^+, *i.e.* the anticlockwise rotation of $\pi/2$ with respect of the z axis.
2. Construct the associated function operator $\hat{R}_{C_4^+}$ and find the transformed function for each of the 5 d atomic orbitals. Find the associated matrix representation of the point symmetry operation in the Hilbert space generated by such orbitals.
3. Repeat the first two steps for all the elements of the cyclic group C_4. Is the representation reducible or irreducible?
4. Analogously, find the matrix representation of the group D_4 in the same Hilbert space.

Frohes Schaffen!