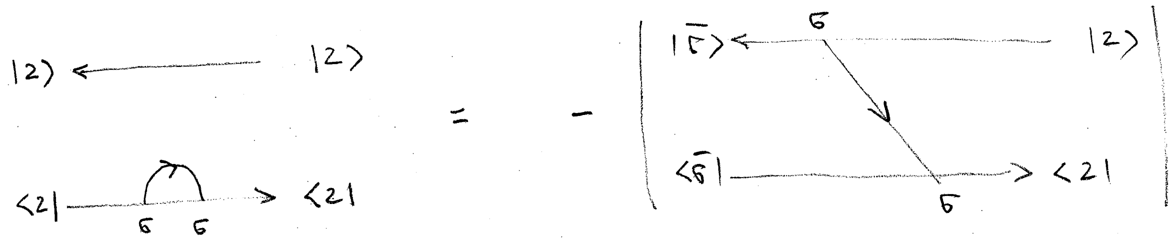
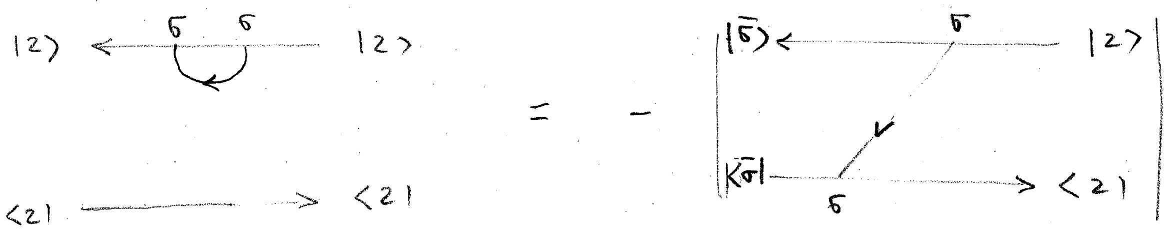
We can thus identify easily the gain-loss pairs



(4.64)

and



(4.65)

The diagrammatic equations above follow immediately from the diagrammatic rules since a change of contour for the last vertex is associated to a change of sign (no change in the number of fermionic line crossing can happen, to every order). $\Rightarrow$ gain-loss pairs of diagrams can be easily found to every order by changing the contour of the first or the last vertex.

4.3.2 Fourth order processes

In this section we identify and classify the fourth order processes from a physical point of view: we denote by

co-tunnelling (COT) the contributions where $\Delta N = 0$

cotunnelling assisted sequential tunnelling $\Delta N = \pm 1$

pair tunnelling $\Delta N = \pm 2$

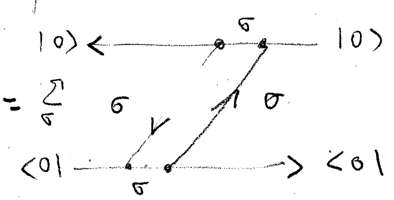
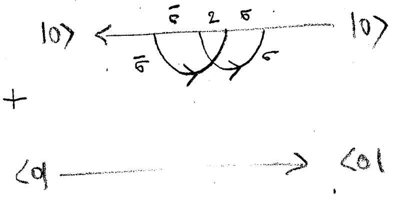
The simple distinction given above should be further complemented with the information that diagrams with a contraction of the first and last time are dressed second order diagrams (see last chapter) while the crossed-contractions have no second order counterpart.

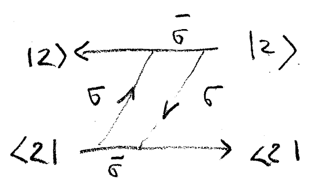
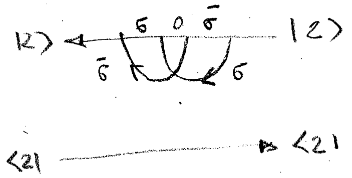
Cotunnelling $\Delta N = 0$

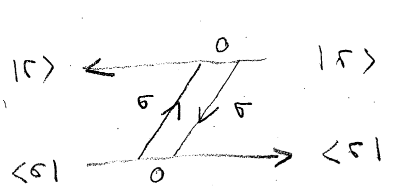
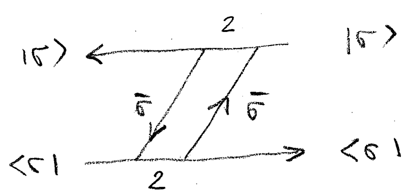
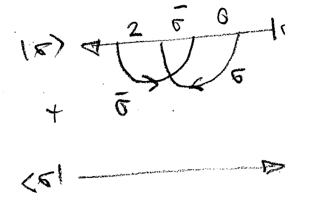
In the AIM we recognize 4 different contributions of cotunnelling processes

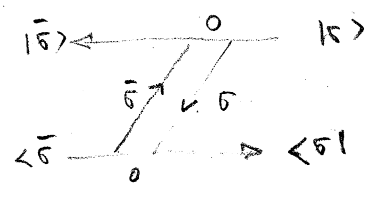
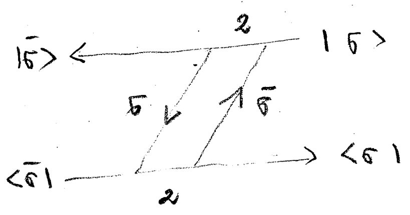
$$|0\rangle \to |0\rangle \;,\quad |\sigma\rangle \to |\sigma\rangle \;,\quad |\sigma\rangle \to |\bar\sigma\rangle \;,\quad |2\rangle \to |2\rangle$$

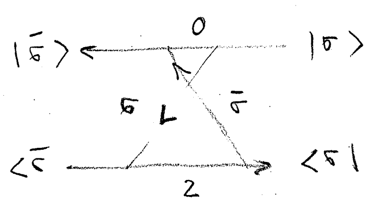
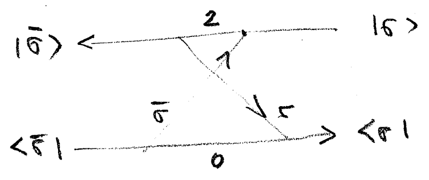
Due to the considerations made above we identify thus the following diagrams:

$\Gamma_{cot}^{0\to 0} = \sum_\sigma$  +  + h.c.

$\Gamma_{cot}^{2\to 2} = \sum_\sigma$  +  + h.c.

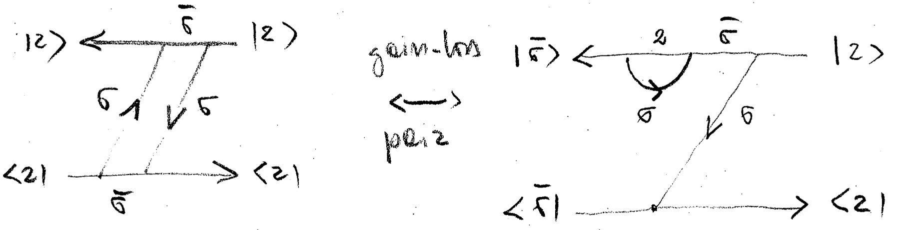
$\Gamma_{cot}^{\sigma\to\sigma} =$  +  +  + h.c.

$\Gamma_{cot}^{\sigma\to\bar\sigma} =$  + 

+  +  + h.c.

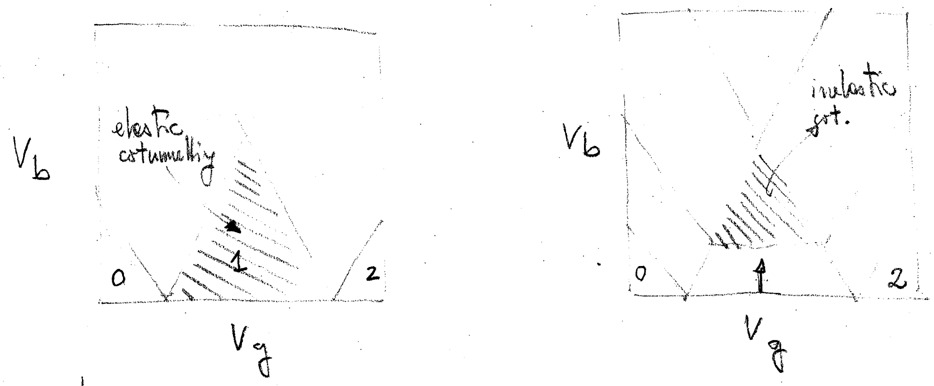
(4.66)

Each of the term above has a counterpart in the gain-loss relation in which the particle number varies by 1: exemplarily



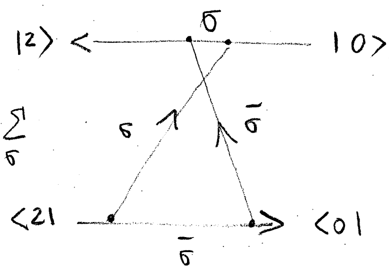
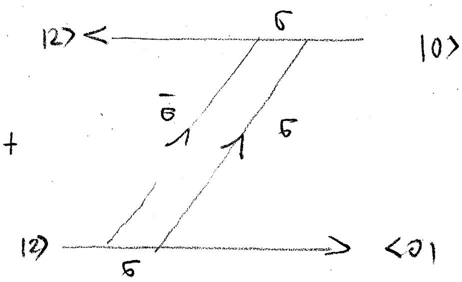
(4.67)

The contribution of cotunnelling are ELASTIC if the initial and final states have the same energy and INELASTIC vice versa (in AIM if $E_\uparrow \neq E_\downarrow$). They give contribution to the conductance. In the first case we obtain a background even at $V_b = 0$. In the second case there is a threshold $eV_b = |E_\uparrow - E_\downarrow|$ which is independent of the gate voltage applied to the system.

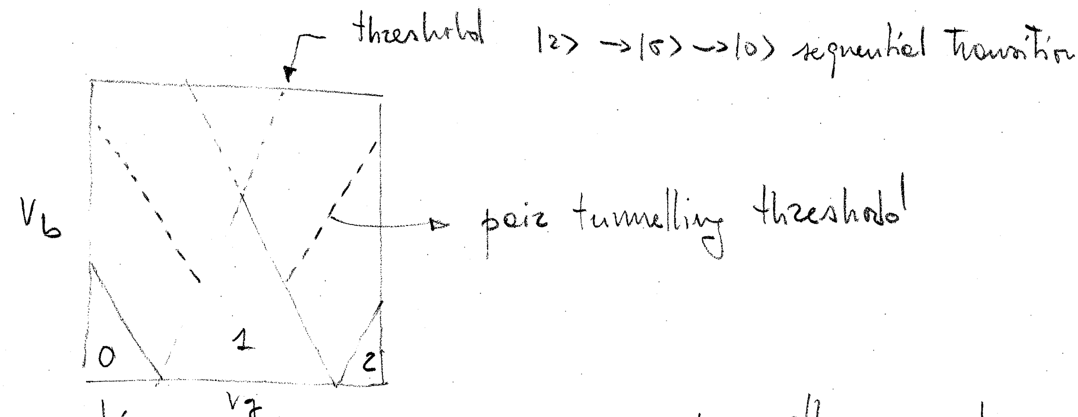


Pair tunnelling $\Delta N = \pm 2$

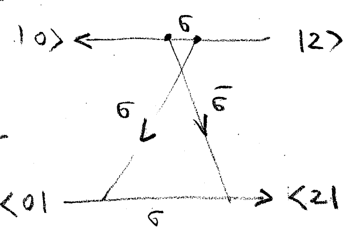
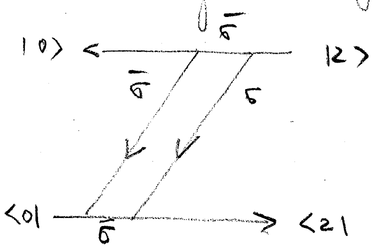
Again within the "crossed" diagrams we consider the ones associated to the coherent transfer of 2 electrons. We have simply to reverse the sign of one fermionic line in (4.66). We obtain

$$\Gamma_{pt}^{0\to 2} = \sum_{\sigma}$$  +  + h.c. (4.68)

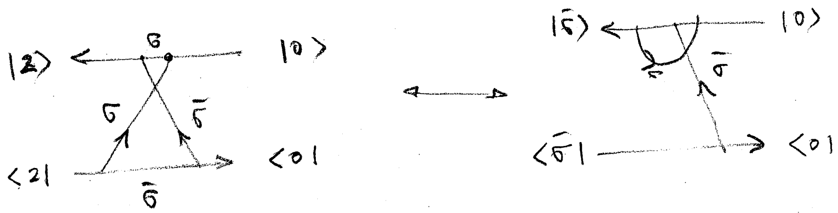
Notice that pair tunnelling processes become energetically allowed at lower biases than the sequential addition/removal $|0\rangle \to |\sigma\rangle \to |2\rangle$ or $|2\rangle \to |\sigma\rangle \to |0\rangle$



The corresponding transitions $|2\rangle \to |0\rangle$ are given by the graphs:

$$\Gamma_{pt}^{2\to 0} = \sum_{\sigma}$$  +  + h.c. (4.69)

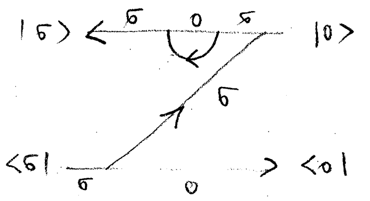
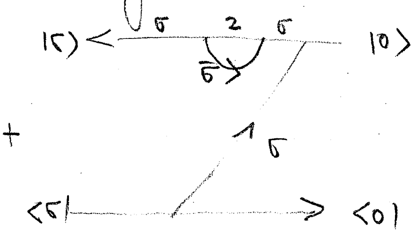
In both cases one can calculate the corresponding gain-loss pairs. For example:

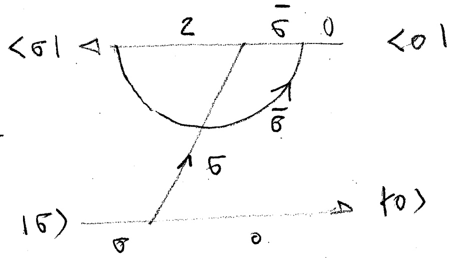
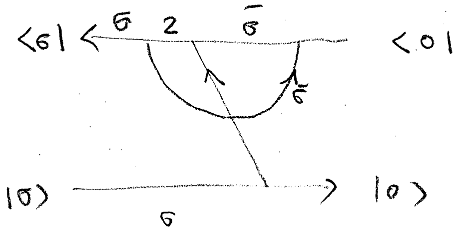


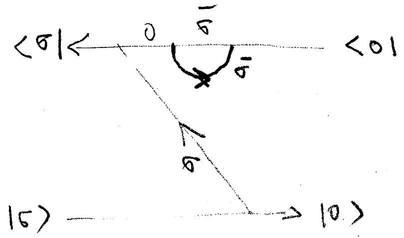
(4.70)

Cotunnelling assisted seq. tunnelling (CO-ST) $\Delta N = \pm 1$

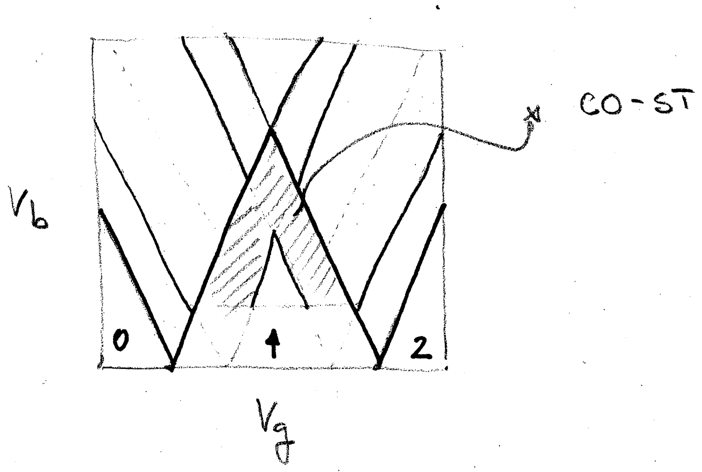
All graphs with cross contracted leads operators are taken by either COT, PT or by their gain-loss partners. All other graphs correspond to renormalization of sequential tunnelling. For example:

$\Gamma^{0 \to \sigma}_{CO-ST} =$  $+$ 

$+$  $+$ 

$+$  $+$ h.c. (4.71)

The cotunnelling assisted sequential tunnelling contributes to transport within the cotunnelling regions, but it is gate dependent. In particular, for the inelastic cotunnelling case one can prove:



4.4 Single time diagrammatics in Liouville space

In the Liouville space representation all operators acting on the RDM superoperatorial level. We introduce to this end the notion of L and R superoperators. Given an operator $\hat{O}$ in Hilbert space

$$\hat{O}_L \hat{X} = \hat{O}\hat{X} \qquad \hat{O}_R \hat{X} = \hat{X}\hat{O} \tag{4.72}$$

Using the definition of the tunnelling Hamiltonian in terms of $\hat{c}$ and $\hat{d}$ operators it is possible to write ($\hat{D}^p_b = \sum_i t^p_{il\vec{k}\sigma} \hat{d}^p_{i\sigma}$)

$$\mathcal{L}_T = -\frac{i}{\hbar} \sum_{\substack{\alpha=L,R \\ bp}} \alpha p \left(\hat{c}^p_b \hat{D}^{\bar{p}}_b \right)_\alpha \qquad \begin{matrix} \alpha = L = 1 \\ \alpha = R = -1 \end{matrix} \tag{4.73}$$

where $b = l, \vec{k}, \sigma$ $\quad l$ = lead index.

Remembering now that the operators on the leads and on the system can be separated at no price, we can write immediately the fundamental component of the theory, the 2nd order Kernel in Schr. picture

$$\wp \mathcal{L}_T \tilde{\mathcal{G}}_0(0) \mathcal{L}_T \wp = -\frac{1}{\hbar^2} \sum_{\{b_i, p_i, \alpha_i\}} \int_0^\infty d\tau \prod_i \alpha_i p_i \, \wp \left(\hat{c}^{p_1}_{b_1} \hat{D}^{\bar{p}_1}_{b_1} \right)_{\alpha_1} \mathcal{G}_0(\tau,0) \left(\hat{c}^{p_0}_{b_0} \hat{D}^{\bar{p}_0}_{b_0} \right)_{\alpha_0} \wp$$

$$= -\frac{1}{\hbar^2} \int_0^\infty d\tau \sum_{b,p,\{\alpha_i\}} \prod_i \alpha_i \, \wp \, \hat{c}^p_{b,\alpha_1} \mathcal{G}_B(\tau,0) c^{\bar{p}}_{b,\alpha_0} \wp \, \hat{D}^{\bar{p}}_{b,\alpha_1} \mathcal{G}_S(\tau,0) D^p_{b,\alpha_0} \tag{4.74}$$

where $\mathcal{G}_B(\tau,0) = e^{\mathcal{L}_B \tau}$ and $\mathcal{G}_S(\tau,0) = e^{\mathcal{L}_S \tau}$. We can continue a step further by noticing

$$\wp \hat{c}^p_{b,\alpha_1} \mathcal{G}_B(\tau,0) \hat{c}^{\bar{p}}_{b,\alpha_2} \wp = \mathcal{G}_B(\tau,0) \wp \hat{c}^p_{b,\alpha_1}(\tau) \hat{c}^{\bar{p}}_{b,\alpha_0} \wp =$$

$$= \wp \hat{c}^p_{b,\alpha_1}(\tau) \hat{c}^{\bar{p}}_{b,\alpha_0} \wp = e^{ip\frac{\varepsilon_b \tau}{\hbar}} f^{(p\alpha_0)}_b(\varepsilon_b) \wp . \tag{4.75}$$

The last equality is proven directly as follows: in general we can write:

$$\mathcal{P}\, \hat{c}^{\,p_1}_{b_1,\alpha_1}\, \hat{c}^{\,p_0}_{b_0,\alpha_0}\, \mathcal{P} = f^{(p_1\alpha_0)}_{b_1}(\varepsilon_{b_1})\, \delta_{b_1 b_0}\, \delta_{p_1 \bar{p}_0}\, \mathcal{P}$$

$\boxed{1}$ $\delta_{b_1 b_2}$ and $\delta_{p_1 \bar{p}_2}$ are due to particle and energy conservation in each bath separately.

$\boxed{2}$ We calculate the 8 cases.

			p_1	α_2
$\mathcal{P}\, c^+_L c_L \mathcal{P}$	$= \rho\, \mathrm{Tr}(c^+ c \rho)$	$= \rho f^+$	$+$	$+$
$\mathcal{P}\, c^+_L c_R \mathcal{P}$	$= \rho\, \mathrm{Tr}(c^+ \rho c)$	$= \rho f^-$	$+$	$-$
$\mathcal{P}\, c^+_R c_L \mathcal{P}$	$= \rho\, \mathrm{Tr}(c \rho c^+)$	$= \rho f^+$	$+$	$+$
$\mathcal{P}\, c^+_R c_R \mathcal{P}$	$= \rho\, \mathrm{Tr}(\rho c c^+)$	$= \rho f^-$	$+$	$-$
$\mathcal{P}\, c_L c^+_L \mathcal{P}$	$= \rho\, \mathrm{Tr}(c c^+ \rho)$	$= \rho f^-$	$-$	$+$
$\mathcal{P}\, c_L c^+_R \mathcal{P}$	$= \rho\, \mathrm{Tr}(c \rho c^+)$	$= \rho f^+$	$-$	$-$
$\mathcal{P}\, c_R c^+_L \mathcal{P}$	$= \rho\, \mathrm{Tr}(c^+ \rho c)$	$= \rho f^-$	$-$	$+$
$\mathcal{P}\, c_R c^+_R \mathcal{P}$	$= \rho\, \mathrm{Tr}(\rho\, c^+_R c_R)$	$= \rho f^+$	$-$	$-$

Eventually we obtain:

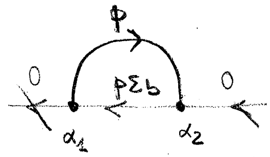
$$\mathcal{P} \mathcal{L}_T \tilde{\mathcal{G}}_0(0) \mathcal{L}_T \mathcal{P} = -\frac{1}{\hbar^2} \int_0^\infty d\tau \sum_{\{\alpha_i\}, p, b} \prod_i \alpha_i\, e^{i p \frac{\varepsilon_b \tau}{\hbar}} f_b^{(p\alpha_2)}\, \mathcal{D}^{\bar{p}}_{b,\alpha_1}\, \mathcal{G}_S(\tau,0)\, \mathcal{D}^{p}_{b,\alpha_0}\, \mathcal{P}$$

$$= +\frac{1}{\hbar^2} \sum_{\{\alpha_i\}, p, b} \mathcal{D}^{\bar{p}}_{b\alpha_1} \frac{f_b^{(p\alpha_2)}}{+\frac{i p \varepsilon_b}{\hbar} + \mathcal{L}_S + \eta} \mathcal{D}^{p}_{b,\alpha_2}\, \mathcal{P} = -\frac{i}{\hbar} \sum_{\{\alpha_i\} b, b} \hat{\mathcal{D}}^{\bar{p}}_{b\alpha_1} \frac{f_b^{(p\alpha_2)}}{p\varepsilon_b - i\hbar \mathcal{L}_S + i\eta} \hat{\mathcal{D}}^{p}_{b\alpha_2}$$

$\lambda \to 0^+$

Tentatively (the diagrammatic rules will be given below in a more precise form).

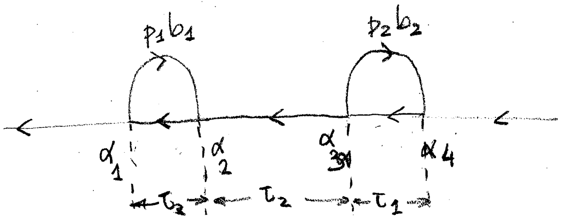
$$\mathcal{P} L_T \tilde{G}_0(0) L_T \mathcal{P} \quad = \tag{4.76}$$



We can now turn to the 4^{th} order contribution

$$\tilde{K}^{(4)}(0) = \mathcal{P} L_T \tilde{G}_0(0) L_T \tilde{G}_0(0) L_T \tilde{G}_0(0) L_T \mathcal{P} - \mathcal{P} L_T \tilde{G}_0(0) L_T \mathcal{P} \tilde{G}_0(0) \mathcal{P} L_T \tilde{G}_0(0) L_T \mathcal{P}$$

The second contribution can be readily interpreted as:



where the propagation between the vertex 3 and 2 is given by G_S

$$\mathcal{P} L_T \tilde{G}_0(0) L_T \mathcal{P} \tilde{G}_0(0) \mathcal{P} L_T \tilde{G}_0(0) L_T \mathcal{P} =$$

$$= -\frac{1}{\hbar^2}\int_0^\infty d\tau_1 \sum_{\substack{b_1 p_1\\ \{\alpha_1,\alpha_2\}}} \alpha_1\alpha_2\, e^{+ip_1\varepsilon_{b_1}\tau_1} f_{b_1}^{(p_1\alpha_2)}(\varepsilon_{b_1})\, D^{\bar{p}_1}_{b_1,\alpha_1} G_S(\tau_1,0)\, D^{p_1}_{b_1,\alpha_2}$$

$$\cdot \int_0^\infty d\tau_2\, G_S(\tau_2,0)\, \cdot$$

$$-\frac{1}{\hbar^2}\int_0^\infty d\tau_3 \sum_{\substack{b_2 p_2\\ \{\alpha_3,\alpha_4\}}} \alpha_3\alpha_4\, e^{ip_2\varepsilon_{b_2}\tau_3} f_{b_2}^{(p_2\alpha_4)}(\varepsilon_{b_2})\, D^{\bar{p}_2}_{b_2,\alpha_3} G_S(\tau_3,0)\, D^{p_2}_{b_2,\alpha_4}\, \mathcal{P}$$

(4.77)

The first contribution can be brought immediately into the form

$$\mathcal{P} L_T \tilde{G}_0(0) L_T \tilde{G}_0(0) L_T \tilde{G}_0(0) L_T \mathcal{P} =$$

$$= \frac{1}{\hbar^4}\int_0^\infty d\tau_1 \int_0^\infty d\tau_2 \int_0^\infty d\tau_3 \sum_{\{b_i,p_i,\alpha_i\}} \prod_i \alpha_i \, \mathcal{P}\, c^{p_3}_{b_3,\alpha_3} G_B(\tau_3,0)\, c^{p_2}_{b_2,\alpha_2} G_B(\tau_2,0)\, c^{p_1}_{b_1,\alpha_1} G_B(\tau_1,0)\, c^{p_0}_{b_0,\alpha_0} \mathcal{P}$$

$$D^{\bar{p}_3}_{b_3,\alpha_3} G_S(\tau_3,0)\, D^{\bar{p}_2}_{b_2,\alpha_2} G_S(\tau_2,0)\, D^{\bar{p}_1}_{b_1,\alpha_1} G_S(\tau_1,0)\, D^{\bar{p}_0}_{b_0,\alpha_0} \mathcal{P}$$

$$= \frac{1}{\hbar^4}\int_0^\infty dt_1 \int_{t_1}^\infty dt_2 \int_{t_2}^\infty dt_3 \sum_{\{b_i,p_i,\alpha_i\}} \prod_i \alpha_i \, \mathcal{P}\, c^{p_3}_{b_3,\alpha_3}(t_3)\, c^{p_2}_{b_2,\alpha_2}(t_2)\, c^{p_1}_{b_1,\alpha_1}(t_1)\, c^{p_0}_{b_0,\alpha_0} \mathcal{P}$$

$$G_S(t_3,0)\, D^{\bar{p}_3}_{b_3,\alpha_3}(t_3)\, D^{\bar{p}_2}_{b_2,\alpha_2}(t_2)\, D^{\bar{p}_1}_{b_1,\alpha_1}(t_1)\, D^{\bar{p}_0}_{b_0,\alpha_0} \mathcal{P}$$

$$\tau_1 = t_1 \qquad \tau_1+\tau_2 = t_2 \qquad \tau_1+\tau_2+\tau_3 = t_3 \qquad (4.78)$$

Now we can apply Wick's theorem to the bath operators in Liouville spa[ce]. The result is the same as with operators but the sign obtaining by crossed contraction only apply to superoperators of the same type. One should use the rule $X_\alpha Y_{\alpha'} = -\alpha\alpha' Y_{\alpha'} X_\alpha$ if $\{X, Y\} = 0$.

$$\mathcal{P}\, c^{p_3}_{b_3,\alpha_3}\, c^{p_2}_{b_2,\alpha_2}\, c^{p_1}_{b_1,\alpha_1}\, c^{p_0}_{b_0,\alpha_0} \mathcal{P} =$$

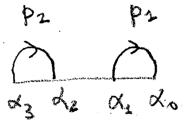
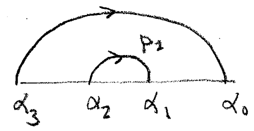
$$\delta_{b_2b_3}\delta_{p_2\bar{p}_3}\delta_{b_1b_0}\delta_{p_1\bar{p}_0}\, \mathcal{P}\, c^{p_3}_{b_3,\alpha_3} c^{p_2}_{b_2,\alpha_2}\mathcal{P}\,\mathcal{P}\, c^{p_1}_{b_1\alpha_1} c^{p_0}_{b_0\alpha_0}\mathcal{P}$$

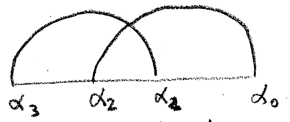
$$-\alpha_1\alpha_2\, \delta_{b_3b_1}\delta_{p_3\bar{p}_1}\delta_{b_2b_0}\delta_{p_2\bar{p}_0}\, \mathcal{P}\, c^{p_3}_{b_3,\alpha_3} c^{p_1}_{b_1\alpha_1}\mathcal{P}\,\mathcal{P}\, c^{p_2}_{b_2\alpha_2} c^{p_0}_{b_0\alpha_0}\mathcal{P}$$

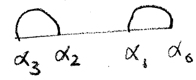
$$+\cancel{\alpha_0^2}\,\alpha_1\alpha_2\, \delta_{b_3b_0}\delta_{p_3\bar{p}_0}\delta_{b_2b_1}\delta_{p_2\bar{p}_1}\, \mathcal{P}\, c^{p_3}_{b_3\alpha_3} c^{p_0}_{b_0\alpha_0}\mathcal{P}\,\mathcal{P}\, c^{p_2}_{b_2\alpha_2} c^{p_1}_{b_1\alpha_1}\mathcal{P}$$

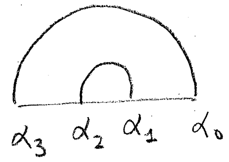
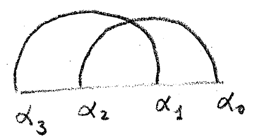
(4.79)

Diagrammatically one thus obtains

$$\mathcal{P} L_T \tilde{G}_0(0) L_T \tilde{G}_0(0) L_T \tilde{G}_0 L_T \mathcal{P} =$$  $+$  $+$

$+$ 

But the first diagram is exactly cancelled by the one associated to $\mathcal{P} L_T G_0 L_T \mathcal{P} \tilde{G}_0(0) \mathcal{P} L_T \hat{G}_0(0) L_T \mathcal{P} =$ . Diagrammatically we can thus write

$$\tilde{K}^{4}(0) =$$  $+$  (4.80)

thus reducing to 2 the 128 diagrams generated in the Hilbert space with the double contour diagrammatics. The time integral in (4.78) can be performed easily and one obtains:

$$\tilde{K}(\lambda)^{(4)} = \frac{1}{\hbar^4} \sum_{\{b_i, p_i, \alpha_i\}} \prod_i \alpha_i$$

$$\alpha_1\alpha_2 \, f_{b_2}^{(p_2\alpha_0)}(\varepsilon_{b_2}) \, f_{b_1}^{(p_1\alpha_1)}(\varepsilon_{b_1}) \, \hat{D}^{\bar{p}_2}_{b_2\alpha_3} \frac{1}{\lambda - L_S - \frac{i p_2 \varepsilon_{b_2}}{\hbar}} \hat{D}^{\bar{p}_1}_{b_1\alpha_2} \frac{1}{\lambda - L_S - \frac{i p_1 \varepsilon_{b_1}}{\hbar} - \frac{i p_2 \varepsilon_{b_2}}{\hbar}}$$

$$\hat{D}^{p_1}_{b_1,\alpha_1} \frac{1}{\lambda - L_S - i p_2 \varepsilon_{b_2}} \hat{D}^{p_2}_{b_2,\alpha_0} +$$

$$-\alpha_1\alpha_2 \, f_{b_2}^{(p_2\alpha_1)}(\varepsilon_{b_2}) \, f_{b_1}^{(p_1\alpha_0)}(\varepsilon_{b_1}) \, \hat{D}^{\bar{p}_2}_{b_2,\alpha_3} \frac{1}{\lambda - L_S - \frac{i p_2 \varepsilon_{b_2}}{\hbar}} \hat{D}^{\bar{p}_1}_{b_1\alpha_2} \frac{1}{\lambda - L_S - i\frac{p_1 \varepsilon_{b_1}}{\hbar} - \frac{i p_2 \varepsilon_{b_2}}{\hbar}}$$

$$\hat{D}^{p_2}_{b_2,\alpha_1} \frac{1}{\lambda - L_S - i p_1 \varepsilon_{b_1}} \hat{D}^{p_1}_{b_1,\alpha_0} \Big] \qquad (4.81)$$

4.4.1 Diagrammatic rules in the superoperator formalism

Ultimately one arrives to the following set of diagrammatic rules for a generic diagram of order $2n$.

i) Draw a propagation line oriented right to left. On it fix $2n$ vertices, each associated to an index α $(= \underset{+}{L}, \underset{-}{R})$

ii) Draw n fermionic lines all oriented from left to right, each labelled with an index p_i and an energy ε_i $(i = 1 \dots n)$ connecting the $2n$ vertices in such a way that the diagram cannot be cut into 2 parts by cutting a single propagator line.

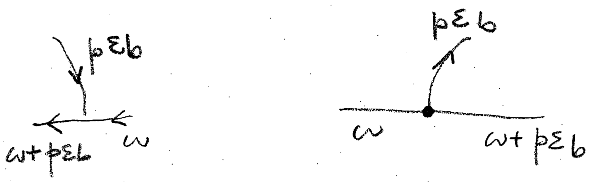
iii) Assign to each fermionic line the number $f_{b_i}^{(p_i \alpha)}(\varepsilon_i)$ where α is the side index of the arriving vertex.

iv) Assign to each vertex a "dressed" system operator $D_{b_i,\alpha}^{p_i}$ or $D_{b_i\alpha'}^{\bar{p}_i}$ respectively for the vertex with ingoing (α) or outgoing (α') fermionic line. Notice that $\hat{D}$ has the dimensions of energy.

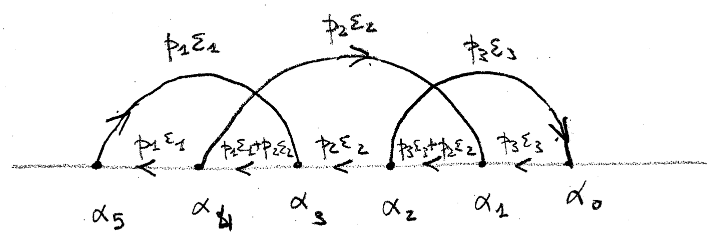
v) Assign to each propagator line between vertices the operator

$$G_{SB}(\omega) = \frac{1}{\omega - i\hbar \mathcal{L}_S + i\eta}$$

vi) The energy in each vertex should be conserved



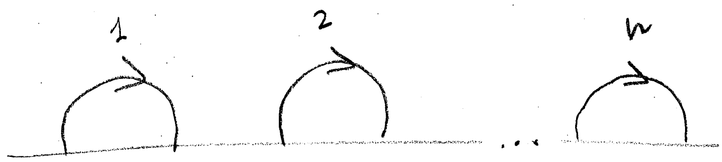
Example of labelled 6^{th} order diagram



Now we turn to the translation of the diagram into a formula

i) Write the product of the vertex operators and the propagators from left to right, respecting the order of the graph

ii) Multiply by a prefactor $\left(-\frac{i}{\hbar}\right)\prod_i \alpha_i\,(-1)^{P(\{\alpha_i\})}$ where $P(\{\alpha_i\})$ is the number of permutation of equal side vertices necessary to recast the graph into a completely reducible form

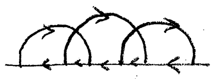


which respects the time ordering of the contraction (the direction of the fermionic lines)

iii) Sum over all internal degrees of freedom: i.e. $\sum_{\{\alpha_i\}}\sum_{\{p_i\}}\sum_{\{b_i\}}$. The last sum can be expressed as

$\sum_{l_i=L,R}\sum_{\sigma_i}\int d\varepsilon_i\,\mathcal{D}_{l_i\sigma_i}(\varepsilon_i)$ where $\mathcal{D}_{l_i\sigma_i}$ is the density of states for the electrons of spin σ_i in the lead l_i.

For the example above we obtain:



$$= -\frac{i}{\hbar}\sum_{\{\alpha_i\}}\sum_{\{p_i\}}\sum_{\{l_i\}}\sum_{\{\sigma_i\}}\int d\vec{\varepsilon}\,\prod_{j=1}^{3}\mathcal{D}_{l_j\sigma_j}(\varepsilon_j)\prod_{i=0}^{5}\alpha_i(-\alpha_4\alpha_3)(-\alpha_2\alpha_1)$$

$$f^{p(p_1\alpha_3)}_{l_1}(\varepsilon_1)\, f^{p(p_2\alpha_1)}_{l_2}(\varepsilon_2)\, f^{p(p_3\alpha_0)}_{l_3}(\varepsilon_3)$$

$$D^{\bar{p}_1}_{l_1\sigma_1,\alpha_5}\frac{1}{p_1\varepsilon_1 - i\hbar\mathcal{L}_s + i\eta}\, D^{\bar{p}_2}_{l_2\sigma_2,\alpha_4}\frac{1}{p_1\varepsilon_1 + p_2\varepsilon_2 - i\hbar\mathcal{L}_s + i\eta}\, D^{p_1}_{l_1\sigma_1,\alpha_3}\frac{1}{p_2\varepsilon_2 - i\hbar\mathcal{L}_s + i\eta}$$

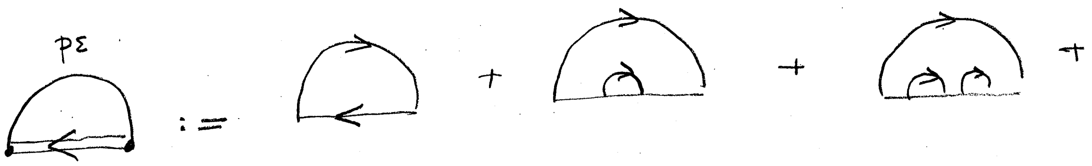
$$D^{\bar{p}_3}_{l_3\sigma_3,\alpha_2}\frac{1}{p_3\varepsilon_3 + p_2\varepsilon_2 - i\hbar\mathcal{L}_s + i\eta}\, D^{p_2}_{l_2\sigma_2,\alpha_1}\frac{1}{p_3\varepsilon_3 - i\hbar\mathcal{L}_s + i\eta}\, D^{p_3}_{l_3\sigma_3,\alpha_0}\,\rho \tag{4.82}$$

4.5 All order resummations: non perturbative effects

Some properties of the transport characteristics through a nanojunction cannot be understood within a perturbative scheme; e.g. the coupling broadened conductance peaks or the Kondo conductance peak for odd particle Coulomb diamonds. There are 2 cases in which the diagrammatic approach presented above allows for resummation to all orders. The dressed second order and the resonant tunnelling approximation.

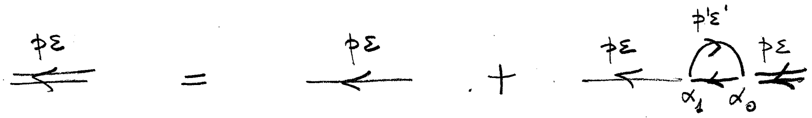
4.5.1 Dressed second order approximation (DSO)

The idea of the DSO is to construct an effective second order kernel containing renormalizations to every order. Diagrammatically



(4.83)

It is clear that the dressed propagator characterizing the DSO approximation can be evaluated by means of a Dyson-like equation



(4.84)

$$G^{DSO}(p\varepsilon) = \frac{1}{p\varepsilon - i\hbar\Delta_s + i\eta} + \frac{1}{p\varepsilon - i\hbar\Delta_s + i\eta} \sum_{\substack{\alpha_1\alpha_0 \\ p'l'}} \int d\varepsilon' \, \mathcal{D}_{l\sigma'}(\varepsilon') \, \Delta^{\bar{p}'}_{l'\sigma',\alpha_1} \frac{f^{(p'\alpha_0)}_{l'}(\varepsilon')}{\varepsilon p + \varepsilon' p' - i\hbar\Delta_s + i\eta} \Delta^{p'}_{l'\sigma',\alpha_1} G^{DSO}$$

The equation above is easily solved in $G^{DSO}(p\varepsilon)$ (it is a linear equation!)

We can, in fact, introduce the DSO self-energy

$$\Sigma^{DSO}(p|\varepsilon) = \sum_{\substack{\alpha_1\alpha_0\\ p'l'}} \int d\varepsilon' \, \mathcal{D}_{l'\sigma'}(\varepsilon') \, \Delta^{\bar{p}'}_{l\sigma',\alpha_1} \frac{f^{(p'\alpha_0)}_{l'}(\varepsilon')}{\varepsilon p + \varepsilon' p' - i\hbar h_s + i\eta} \Delta^{p'}_{l\sigma',\alpha} \tag{4.85}$$

and express:

$$G^{DSO}(p\varepsilon) = \frac{1}{p\varepsilon - i\hbar h_s - \Sigma^{DSO}(p\varepsilon)} \tag{4.86}$$

proof of (4.86)

$$G^{DSO}(p\varepsilon) = G_{SB}(p\varepsilon) + G_{SB}(p\varepsilon)\,\Sigma^{DSO}(p\varepsilon)\,G^{DSO}(p\varepsilon)$$

$$\left(1 - G_{SB}(p\varepsilon)\,\Sigma^{DSO}(p\varepsilon)\right) G^{DSO}(p\varepsilon) = G_{SB}(p\varepsilon)$$

$$G^{DSO}(p\varepsilon) = \left(1 - G_{SB}(p\varepsilon)\,\Sigma^{DSO}(p\varepsilon)\right)^{-1} G_{SB}(p\varepsilon) =$$

$$= \left[\left(G_{SB}(p\varepsilon)\right)^{-1}\left(1 - G_{SB}(p\varepsilon)\,\Sigma^{DSO}(p\varepsilon)\right)\right]^{-1}$$

$$= \left[G_{SB}(p\varepsilon)^{-1} - \Sigma^{DSO}(p\varepsilon)\right]^{-1}.$$

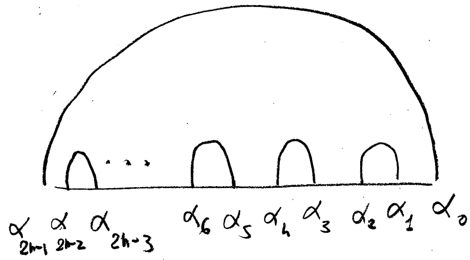
■

The dressed second order Kernel reads:

$$\tilde{K}^{DSO} = -\frac{i}{\hbar} \sum_{\substack{\alpha_1\alpha_0\\ pl\sigma}} \alpha_1\alpha_0 \int d\varepsilon \, \mathcal{D}_{l\sigma}(\varepsilon) \, \Delta^{\bar{p}}_{l\sigma,\alpha_1} \frac{f^{(p\alpha_0)}_{l}(\varepsilon)}{p\varepsilon - i\hbar h_s - \Sigma^{DSO}(p\varepsilon)} \Delta^{p}_{l\sigma,\alpha_0} \tag{4.87}$$

<u>Notice</u>: Σ^{DSO} has <u>no</u> α prefactors since $(-1)^{P(\alpha_i)} = \prod_{i=1}^{2n-2} \alpha_i$ for a diagram

of the type:



The physical meaning of (4.87) rely on the presence of Σ^{DSO} in the denominator of the propagator. In the calculation of the rates (i.e. the different components of $\tilde{k}^{DSO}$) the bias and gate dependence is not given only by the temperature (in f) and the system energy difference (Δ_s) but also by the tunnelling coupling Γ contained in the imaginary part of Σ^{DSO}.