Universität Regensburg Wintersemester 08-09

Quantentheorie II

Prof. Milena Grifoni Room H34

Andrea Donarini Dienstag, 15 Uhr c.t.
Benedikt Scharf Donnerstag, 13 Uhr c.t.
Juan-Diego Urbina Mittwoch, 13 Uhr c.t.
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Blatt 1

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1. Perturbation theory of a four-level system

Let us consider the quantum ring described by the four normalized and orthogonal states $|1\rangle$, $|2\rangle$, $|3\rangle$ and $|4\rangle$ with the associated Hamilton operator:

$$H = \sum_{i=1}^{4} (-1)^{i-1} \varepsilon\, |i\rangle\langle i| - \Delta\big(\, |i\rangle\langle i+1| + |i+1\rangle\langle i|\,\big)$$

with the periodic condition $|5\rangle \equiv |1\rangle$.

a) • Calculate the exact eigenenergies for the system by diagonalizing the matrix corresponding to the given Hamiltonian. Make the Taylor expansion of the eigenenergies to second order in the parameter $\frac{\Delta}{\varepsilon}$. (**2 Points**)

b) • Rewrite the Hamiltonian in the basis

$$|\alpha\rangle = \frac{1}{\sqrt{2}}(\,|1\rangle + |3\rangle)\,, \qquad |\beta\rangle = \frac{1}{\sqrt{2}}(\,|2\rangle + |4\rangle)\,,$$
$$|\gamma\rangle = \frac{1}{\sqrt{2}}(\,|1\rangle - |3\rangle)\,, \qquad |\delta\rangle = \frac{1}{\sqrt{2}}(\,|2\rangle - |4\rangle)\,,$$

and prove that the pairs of vectors $\{\,|\alpha\rangle, |\beta\rangle\}$ and $\{\,|\gamma\rangle, |\delta\rangle\}$ generate two subspaces with independent dynamics. Can you say why? Calculate the exact eigeneneregies and compare with the result at point a). Calcualte the exact eigenstate for the system. (**4 Points**)

c) • Consider the situation $0 < \Delta \ll \varepsilon$. In the basis introduced at point b) calculate the eigenenergies of the system with the help of the non-degenerate perturbation theory. Keep terms up to $\frac{\Delta^2}{\varepsilon}$. Compare these energies with the ones calculated at point a). Sketch the spectrum of the Hamiltonian as a function of Δ for fixed ε. (**4 Points**)

2. Ritz's variation principle

a) Calculate, with the help of the variational principles, the upper limit for the ground-state energy of a particle of mass m in a three-dimension Coulomb potential

$$V(\vec{r}) = -\frac{Z}{r}$$

with $r = |\vec{r}|$ and $Z > 0$.

Start with the variational Ansatz

$$\psi_\alpha(\vec{r}) = A\mathrm{e}^{-\alpha r}\,.$$

b) Use the variational principle to prove that a one dimension binding potential always admits one bound state. (*Hint*: Show that $\langle\Psi| H |\Psi\rangle$ can always be made negative by choosing Ψ as an appropriate test function, for example $N\mathrm{e}^{-\beta^2 x^2}$.)

Frohes Schaffen!

Return the solution of the exercises marked with • by Monday 20th of October at 10:00.