Universität Regensburg Wintersemester 08-09

Quantentheorie II

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Blatt 3

1. Operators in the Heisenberg picture

In the Heisenberg picture the states $|\psi_{\rm H}\rangle$ of a system are stationary while the observables $A_{\rm H}$ are time dependent and they evolve according to the equation

$$i\hbar\frac{dA_{\rm H}}{dt} = [A_{\rm H}(t), H(t)]\,,$$

where $H(t)$ describes the Hamiltonian of the system (in general explicitly time dependent).

a) Show that the position and momentum operators satisfy, in both the Schrödinger and Heisenberg picture, the same commutator relations:

$$[X_{\rm H}(t), P_{\rm H}(t)] = [X_S, P_S] = i\hbar\,, \qquad \forall t.$$

b) Prove that, for an Hamiltonian of the form

$$H = \frac{P^2}{2m} + V(X,t)\,,$$

the following equations hold (Ehrenfest theorem):

$$\frac{\rm d}{{\rm d}t}X_{\rm H}(t) = \frac{1}{m}P_{\rm H}(t)\,,$$
$$\frac{\rm d}{{\rm d}t}P_{\rm H}(t) = -\frac{\partial V}{\partial x}(X_{\rm H}(t),t)\,.$$

c) • Specialize the equations derived at point b) to the case of a harmonic oscillator and, by solving them, give explicitly the time evolution of the position and momentum operators $X_{\rm H}$ and $P_{\rm H}$ in the Heisenberg picture. Which are the initial conditions for the problem? (**3 Points**)

d) • Consider a harmonic oscillator which at time $t = 0$ is prepared in the state described by the wave function:

$$\psi(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} \exp\left[-\frac{m\omega}{2\hbar}(x-x_0)^2\right].$$

Calculate the time evolution of the expectation values $\langle X_{\rm H}\rangle$ and $\langle P_{\rm H}\rangle$. Make a sketch of their time dependance. (**3 Points**)

2. Time-dependent perturbations

Consider a one-dimensional simple harmonic oscillator whose classical angular frequency is ω_0. For $t < 0$ it is known to be in the ground state. For $t \geq 0$ the system is perturbed.

a) Let the perturbation be the time dependent but spatially uniform force:

$$F(t) = F_0 \cos(\omega t),$$

where F_0 is constant both in space and time. Obtain an expression for the expectation value $\langle x \rangle$ as a function of time using time-dependent perturbation theory to lowest nonvanishing order. Is this procedure valid for $\omega \simeq \omega_0$?

b) • Consider now the perturbation given by the force:

$$F(t) = F_0 \exp(-\tfrac{t}{\tau}).$$

Using time-dependent perturbation theory to first order, obtain the probability of finding the oscillator in its first excited state for $t > 0$. Show that the $t \to \infty$ (τ finite) limit of your expression is independent of time. Is this reasonable or surprising? Can we find higher excited states? (**4 Points**)

Hint: You may use

$$\langle n|x|n' \rangle = \sqrt{n}\delta_{n',n-1} + \sqrt{n+1}\delta_{n',n+1}.$$

Frohes Schaffen!

Return the solution of the exercises marked with • by Monday 3rd of November at 10:00.