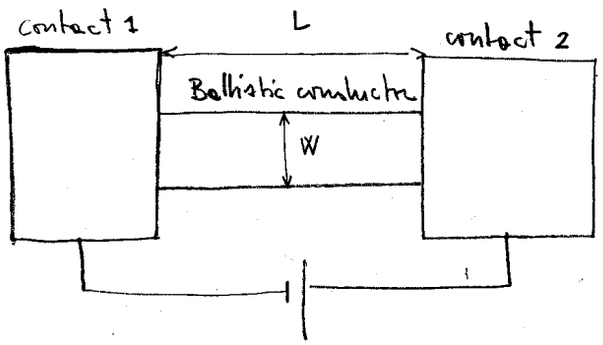
CONDUCTANCE FROM TRANSMISSION

The basic idea due to R. Landauer (19) is to relate the current through a conductor to the probability that an electron is transmitted through it. (let's call T this probability)

If $T \ll 1$ low current => high resistance

$T \sim 1$ current is FINITE : what is giving the resistance ?

— RESISTANCE OF A BALLISTIC CONDUCTOR



The conductance $G = \sigma \frac{W}{L}$. $L \to 0$ ($L \ll L_m$ = mean free path) $G \to \infty$?

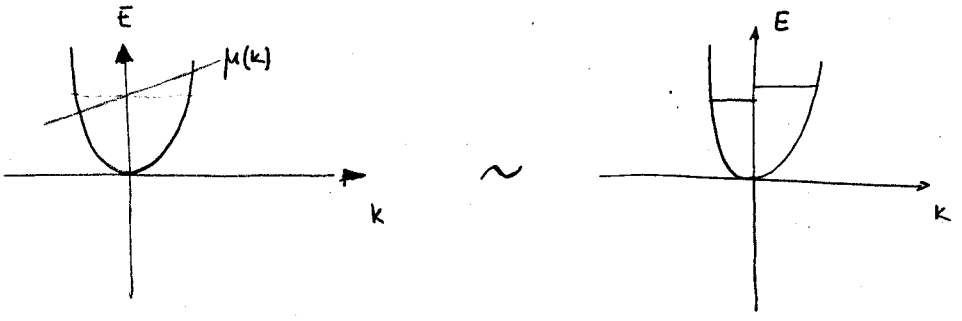
The answer is NO: G saturates to G_c. Notice that this is not a quantum mechanical effect in the sense that we can assume $L_\varphi \ll L$.

G_c = contact resistance

It arises from the redistribution of the current between the nearly ∞ transverse modes of the leads and the finite small number of modes of the ballistic conductor.

Recall the concept of "local (in k) chemical potential" to explain the existence of current in the Boltzmann formalism.

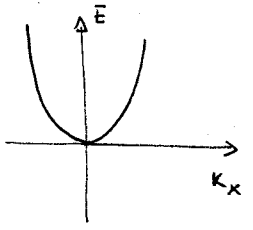
Non-equilibrium distribution



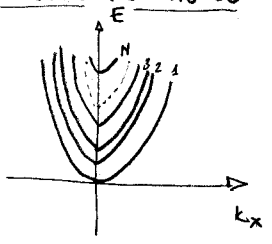
The basic idea is an unbalance between the forward and the backward movers that gives a current.

Dispersion relations:

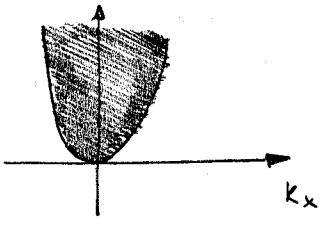
1 transverse mode



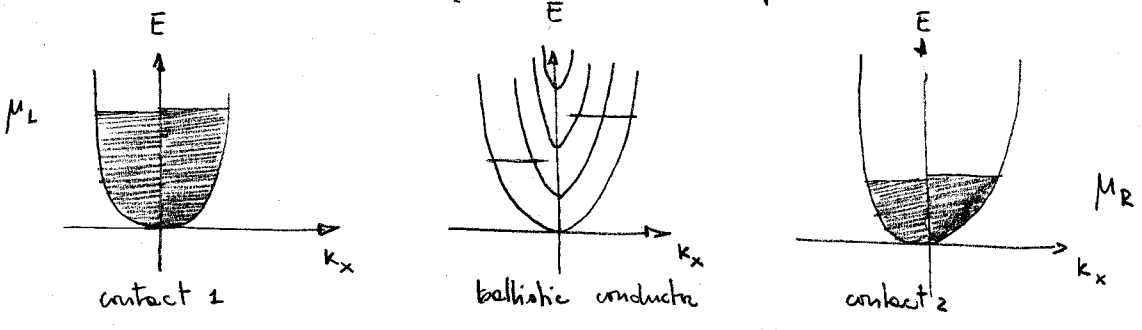
N - transverse modes



∞ - transverse modes



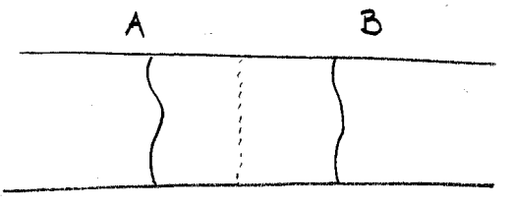
The qualitative situation for our sample is:



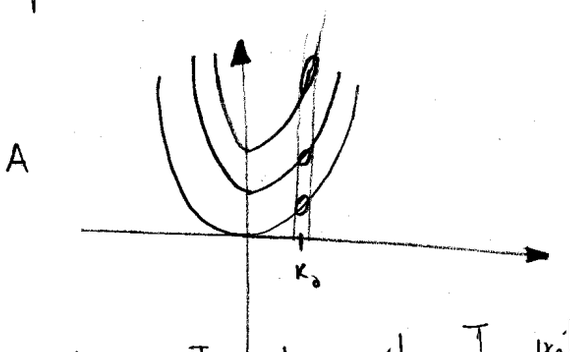
Each transverse mode can carry a finite amount of current. If we want a given current to flow from left to right in the sample it is enough to have a very small imbalance in the levels that govern a very high density of transverse modes while we must have a large imbalance in the mode occupations of the conductor that has a very low density of transversal modes.

In order to calculate the contact resistance G_c^{-1} we consider a ballistic conductor and calculate the current through it for a given applied bias $\mu_L - \mu_R$. In order to proceed we introduce the concept of <u>reflectionless contact</u>

Let us consider a finite width conductor artificially separated in 2 regions A, B

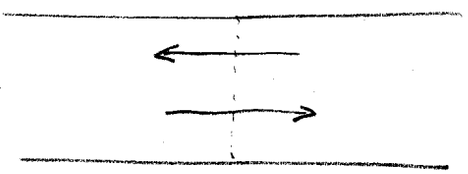


The states in A and B can be classified in (n, k_x) where n labels the transversal mode and k_x the x direction wave number. By calculation of the group velocity we can distinguish among right movers ($k_x > 0$) and left movers ($k_x < 0$). Now let's construct a wave packet in A centered around $k_0 > 0$. I can do it in many different ways due to the presence of transverse modes

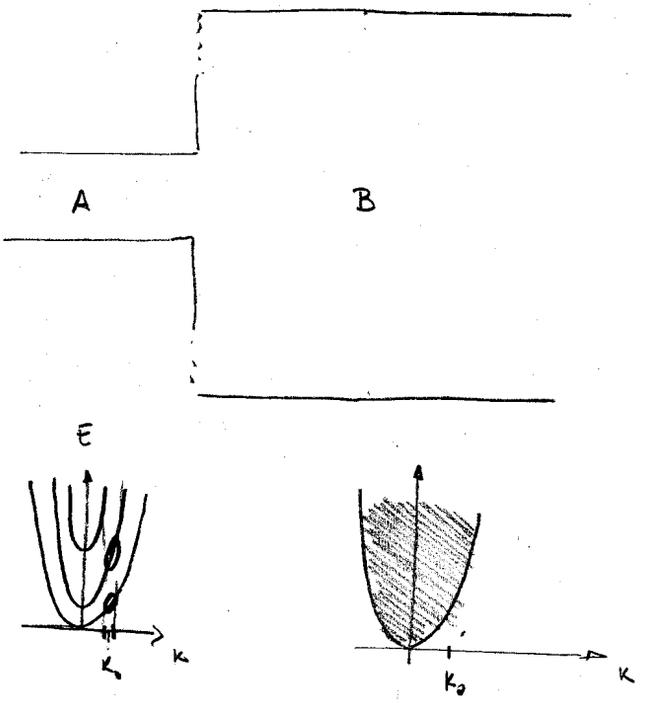


Different states around k_0 available for the construction of the wave packet.

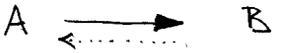
No matter how I do it I will always find a companion state in B with the same transverse structure in which to scatter... The "overlap" at the interface is perfect. This fictitious junction is reflectionless in both directions.



Now let us try the same thought experiment with the junction



For a given k_0 I have a finite number of transverse bands in A that I will certainly encounter in B. But the viceversa is not true. Most of the wave packets that I construct in B involve transverse modes unknown in A => these travelling wave packets will be reflected at the interface. We can represent it as



i.e. the junction acts as a complete filter.

Now we are ready to go back to our contact | conductor | contact structure

Notice that current can flow trough a reflectionless system like the one just sketched. One has to consider the electrons that can enter the BC. +k states up to μ_L, -k up to μ_R. Thanks to the reflectionless contacts +k and -k states cannot mix in BC. (of course also thanks to the absence of scattering centers)

In fact we are just saying that the left movers ($-k$) in the BC will be in "equilibrium" with the left movers in C_R, i.e. at chemical potential μ_R. The right movers with μ_L. The current will be given (at least at 0 temperature) by the states occupied between μ_L and μ_R.

CALCULATING THE CURRENT

Let us call $\varepsilon_N = E(N, k=0)$ the bottom of the N^{th} subband. The number of modes having an energy smaller than $\bar{E}$:

$$M(\bar{E}) = \sum_N \theta(\bar{E} - \varepsilon_N)$$

If we know the current carried by each transverse mode, then we can simply sum over the channels. Consider now a transverse mode whose the states are occupied according to $f^+(\bar{E})$. $I = env$

$$I^+ = e \frac{1}{L} \sum_k v f^+(E) = \frac{e}{L} \sum_k \frac{1}{\hbar} \frac{\partial E}{\partial k} f^+(\bar{E})$$

$$\sum_k \longrightarrow 2_s \frac{L}{2\pi} \int dk$$

$$I^+ = \frac{e}{L} \frac{2L}{2\pi} \int_0^\infty dk \frac{1}{\hbar} \frac{\partial E}{\partial k} f^+(E) = \frac{2e}{h} \int_\varepsilon^\infty dE\, f^+(E)$$

N.B. $\frac{\partial E}{\partial k}$ is exactly the Jacobian of the transformation

$$= \frac{2e}{h} \int_{-\infty}^{+\infty} dE\, f^+(\bar{E})\, \theta(E - \varepsilon).$$

The extension to a multimode (in absence of intermode scattering)

$$I^+ = \sum_N \frac{2e}{h} \int_{-\infty}^{+\infty} f^+(E)\, \theta(E - \varepsilon_N) = \frac{2e}{h} \int_{-\infty}^{+\infty} f^+(\bar{E})\, M(\bar{E})$$

– CONTACT RESISTANCE

If now we assume M constant over the energy range $\mu_L > E > \mu_R$

$$I = \frac{2e^2}{h} M \frac{\mu_L - \mu_R}{e} \Rightarrow G_c = \frac{2e^2}{h} M$$

Thus the contact resistance $G_c^{-1} = \frac{h}{2e^2 M} \sim \frac{12.9\ k\Omega}{M}$.

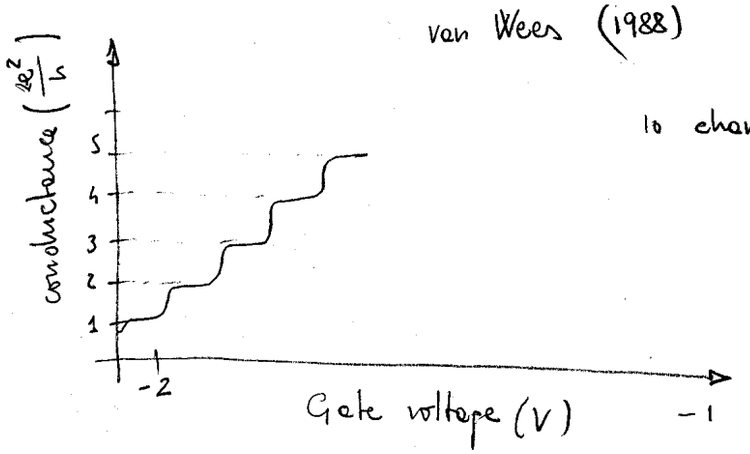
Estimate M for a wide channel:

$$\Delta k_y = \frac{2\pi}{W} \quad \text{at} \quad E_f = \frac{\hbar^2 k_f^2}{2m} \qquad k_y \in [-k_f, k_f]$$

$$\Rightarrow \quad M = \text{Int}\left[\frac{2k_f W}{2\pi}\right] = \text{Int}\left[\frac{2W}{\lambda_f}\right] \quad \Rightarrow \quad \begin{matrix} W = 15\,\mu m \\ \lambda_f = 30\,nm \end{matrix} \quad M \sim 1000.$$

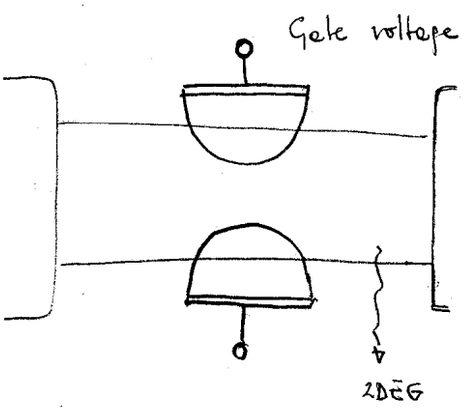
EXPERIMENTAL REALIZATION : <u>QPC</u> quantum point contact

van Wees (1988)



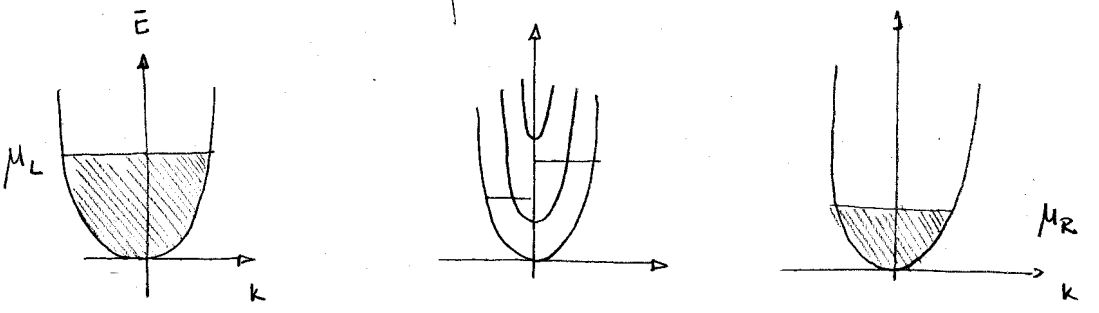
10 channels

the gate voltage controls the width of the trasport "channel".



- WHERE IS THE VOLTAGE DROP?

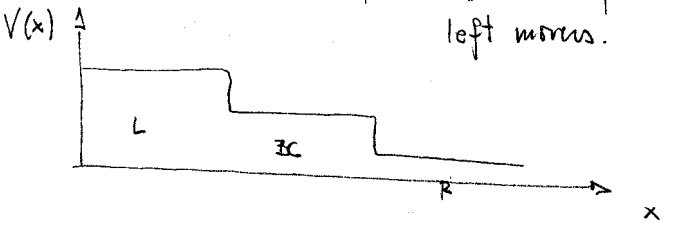
The idea is to associate the voltage to the average chemical potential of the electrons. => it follows that



$$V(x) = \frac{\mu_+ + \mu_-}{2}$$

μ_+ = chemical potential of right movers.

μ_- = chemical potential of left movers.

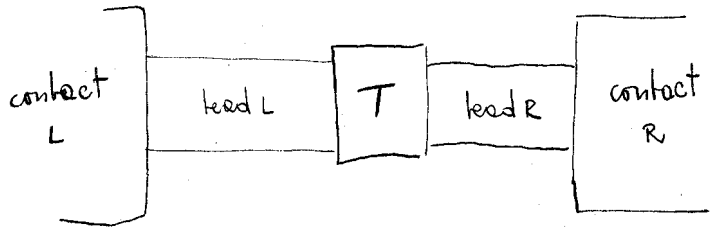


It results that the voltage drop is at the interface between contact and conductor.

Alternatively one could say $V(x) = \mu_+$ or $V(x) = \mu_-$ with similar results. The definition involving the average is the one that gets closer to the alternative one in terms of the electrostatic potential generated by the charge distribution.

- LANDAUER FORMULA

$$G = \frac{2e^2}{h} MT$$



The Landauer formula is saying that, in presence of reflectionless contacts, and of leads with M occupied transverse modes, and finally of some scattering center of transmission probability T => $G = \frac{2e^2}{h} MT$.

N.B. In the Landauer formula the contact resistance has already been taken into account. Let us derive it

$$I_1^+ = \frac{2e}{h}\int_{-\infty}^{+\infty} M f_L^+ dE \quad \text{thanks to the reflectionless contact } L$$

$$I_1^- = -\frac{2e}{h}\int_{-\infty}^{+\infty} MT f_R^- dE - \frac{2e}{h}\int M(1-T) f_L^+ \quad \text{due to the reflectionless contact + the scattering center } T.$$

$$I_1 = \frac{2e}{h}\int_{-\infty}^{+\infty} MT\,(f_L^+ - f_R^-) \overset{\substack{T\to 0 \\ \text{comp} \\ \text{M constant}}}{=} \frac{2e}{h} MT(\mu_L - \mu_R)$$

check of current conservation

$$I_2^+ = \frac{2e}{h}\int MT\, f_L^+ dE + \frac{2e}{h}\int M(1-T) f_R^- dE$$

$$I_2^- = -\frac{2e}{h}\int M f_R^-$$

$$I_2 = \frac{2e}{h}\int MT\,(f_L^+ - f_R^-)\,dE \simeq \frac{2e}{h} MT(\mu_L - \mu_R)$$

$$\Rightarrow \quad I = I_1 = I_2 = \frac{2e}{h} MT\,(\mu_L - \mu_R) \quad \Rightarrow \quad G = \frac{2e^2}{h} MT$$

Should we include the contacts in the calculation? One can interpret the contact-lead interface as a peculiar scattering barrier:

$$I = \frac{2e}{h}(\mu_L - \mu_R)\, M_W \cdot \frac{M}{M_W}$$

M_W is the number of modes in the contacts while $\frac{M}{M_W}$ is the average transmission probability per mode. One can get to this result with the previous considerations. $\Rightarrow$ I is independent from the position and, for practical reasons is better to calculate it within the leads.

Ohm's law

In the Landauer formula
- length independent interface resistance
- the discrete steps associated to the # of transverse modes.

Is Ohm's law also contained? We have to take a wide conductor. The # of modes is proportional to the width:

$$M \cong \frac{k_F W}{\pi} \qquad \left(\frac{2k_F}{\Delta k} = \frac{k_{max} - k_{min}}{\Delta k} \right)$$

$$G = \frac{2e^2}{h} MT = \frac{\cancel{2}e^2}{h} \frac{k_F W}{\cancel{\pi}} T \cdot \frac{m}{m} \frac{\hbar}{\hbar} \cdot \frac{\cancel{2}\pi}{\cancel{2}\pi}$$

$$= \frac{e^2}{\hbar^2} \cdot \frac{k_F W}{\pi^2} \frac{\hbar}{m} \cdot m = e^2 \left(\frac{\hbar k_F}{m} \right) \left(\frac{m}{\pi \hbar^2} \right) \cdot \frac{W}{\pi} T =$$

$$= e^2 W \frac{v_F T}{\pi} N_s$$

(v_F: $\frac{\hbar k_F}{m}$; N_s: $\frac{m}{\pi \hbar^2}$)

Now we have to analyze which is the transmission probability through a conductor of length L:

We start with two scatterers in series T_1, T_2. Naively one would say: $T_{12} = T_1 T_2$ but this result is WRONG since it does not take into account multiple scattering events

$T_1 T_2$ direct path

$T_1 T_2 R_1 R_2$ one bounce

$T_1 T_2 (R_1 R_2)^2$ two bounces

$$T_{12} = \frac{T_1 T_2}{1 - R_1 R_2}$$

Interestingly:

$$\frac{1-T_{12}}{T_{12}} = \frac{1 - \frac{\bar{T}_1\bar{T}_2}{1-R_1R_2}}{\frac{T_1T_2}{1-R_1R_2}} \cdot (1-R_1R_2) =$$

$$= \frac{1 - R_1R_2 - \bar{T}_1\bar{T}_2}{\bar{T}_1\bar{T}_2} = \frac{1 - (1-T_1)(1-T_2) - \bar{T}_1\bar{T}_2}{T_1T_2} =$$

$$= \frac{\cancel{1} - \cancel{1} + T_1 + T_2 - 2\bar{T}_1\bar{T}_2}{\bar{T}_1\bar{T}_2} = \frac{1-\bar{T}_1}{\bar{T}_1} + \frac{1-\bar{T}_2}{\bar{T}_2}$$

which means that $\frac{1-\bar{T}}{T}$ is the quantity characterized by series additivity.

If now we assume N equal scatterers and the associated transmission probability $T(N)$.

$$\frac{1-T(N)}{T(N)} = N\,\frac{1-\bar{T}}{\bar{T}} \Rightarrow T(N) = \frac{T}{N(1-\bar{T}) + T}$$

If we introduce now the density of scatterers $\nu \Rightarrow N = \nu L$

$$T(L) = \frac{T}{\nu L(1-\bar{T}) + T} = \frac{T}{\nu(1-\bar{T})} \cdot \frac{1}{L + \frac{T}{\nu(1-\bar{T})}} = \frac{L_0}{L + L_0}$$

But

$1-\bar{T}$ is the probability of being reflected (= scattered in 1D)

$\frac{1}{\nu}$ = average distance between scatterers

$\frac{1}{1-\bar{T}} \cdot \frac{1}{\nu} \cong L_m$ in fact if the probability of being scattered goes to 0 $\Rightarrow L_m \to \infty$.

$L_m \cong L_0$

Returning to the conductance

$$G = e^2 W \frac{v_F}{\pi} \frac{L_0}{L+L_0} N_s \qquad \frac{v_F L_0}{\pi} \sim D \qquad D = \frac{v_F L_m}{d}$$

$$\simeq e^2 W \frac{D}{L+L_0} N_s = \sigma \frac{W}{L} \cdot \frac{L}{L+L_0}$$

In other terms:

$$G^{-1} = \frac{L}{W}\sigma^{-1} + \frac{L_0}{W}\sigma^{-1}$$

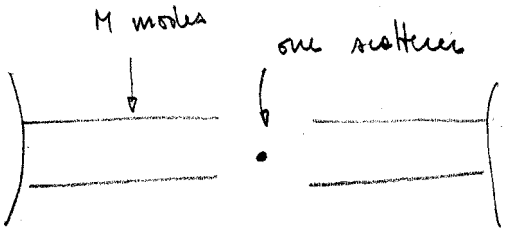
actual resistance — contact resistance.

$$\frac{L_0}{W}\sigma^{-1} = \frac{1}{W} \cdot L_0 \cdot \frac{1}{e^2} \cdot \frac{1}{D} \cdot \frac{1}{N_s} =$$

$$= \frac{1}{W} \cancel{L_0} \frac{1}{e^2} \cdot \frac{\pi}{v_F \cancel{L_0}} \cdot \frac{\pi \hbar^2}{m} = \frac{1}{W} \frac{1}{e^2} \cancel{\frac{\pi m}{\hbar k_F}} \cancel{\frac{\pi \hbar^2}{m}}$$

$$= \frac{1}{W} \frac{2\pi}{2k_F} \frac{h}{2e^2} \simeq \frac{h}{2e^2} \frac{1}{M} = G_c^{-1}.$$

WHERE IS THE RESISTANCE



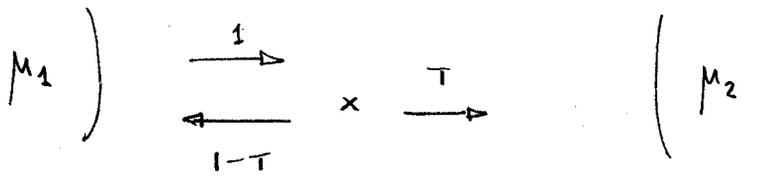
$$G = \frac{2e^2}{h} M T$$

$$G^{-1} = \frac{h}{2e^2 M} \frac{1}{T} = \frac{h}{2e^2 M} + \frac{h}{2e^2 M} \frac{1-T}{T}$$

contact resistance — actual resistance.

? Where is the energy dissipated associated to the "actual resistance", on the scatterer?

In order to answer to the question about the dissipation we have to analyze the local energy distribution of the electrons



LEFT of x:

$$f^+(E) = \theta(\mu_1 - E)$$

$$f^-(E) = T\,\theta(\mu_2 - E) + (1-T)\,\theta(\mu_1 - E) \qquad \text{near } x$$

$$f^-(E) \cong \theta(F' - E) \qquad \text{far from } x \text{ energy relaxation.}$$

F' is calculated by conservation of left movers between near and far left

$$N^- = T\mu_2 + (1-T)\mu_1 = \mu_2 + (1-T)(\mu_1 - \mu_2) = F'$$

[We set the bottom of the band to 0. Whatever energy can be taken as 0.]

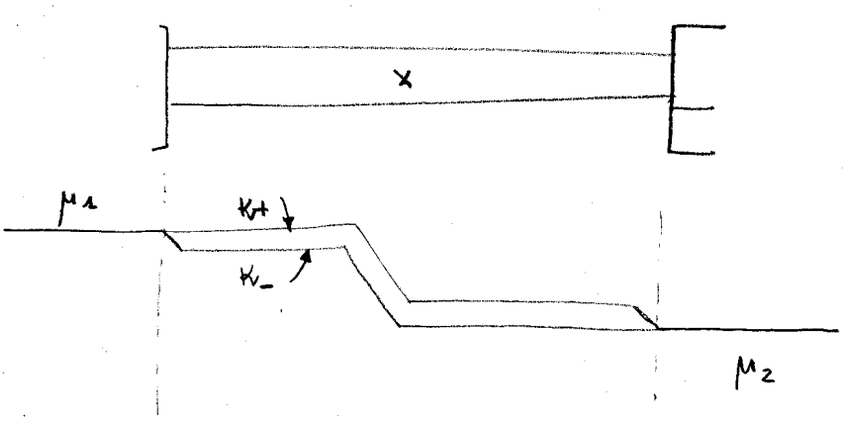
RIGHT of x:

$$f^-(E) = \theta(\mu_2 - E)$$

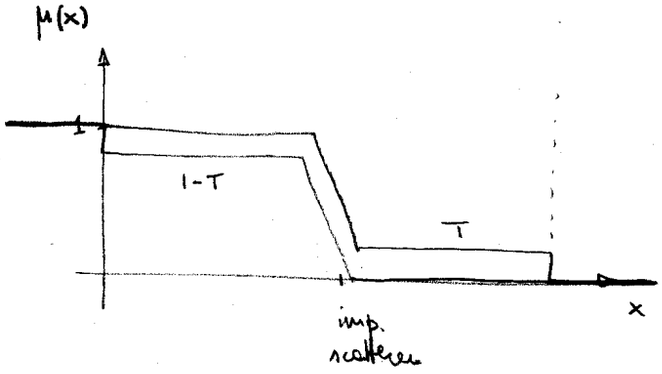
$$f^+(E) = T\,\theta(\mu_1 - E) + (1-T)\,\theta(\mu_2 - E) \qquad \text{near } x$$

$$f^+(E) \cong \theta(F'' - E) \qquad \text{far from } x \text{ energy relaxation}$$

$$F'' = T\mu_1 + (1-T)\mu_2 = \mu_2 + T(\mu_1 - \mu_2)$$



If we introduce the normalized electrochemical potential $\mu_1 = 1$ $\mu_2 = 0$



$V = IR$

The potential drop at the impurity is thus $eV_s = (1-T)(\mu_1 - \mu_2)$. Since the current associated to these potential drops is $I = \frac{2e}{h} MT(\mu_1 - \mu_2)$ $\Rightarrow$ the scatterer has exactly a resistance $G_s^{-1} = \frac{h}{2e^2 M} \frac{1-T}{T}$ in series with the contact resistance $G_c^{-1} = \frac{h}{2e^2 M}$.

HEAT DISSIPATION AND ENERGY DISTRIBUTION FOR THE CURRENT

$P_D = RI^2$ + assume a rigid and elastic scatterer. $\Rightarrow$ dissipation via phonon generation

$P_D = -\frac{d}{dx} I_U$ I_U is the energy current. If particles flowing at high energies start to flow at low energies, in that point there is dissipation.

$I_U = \frac{1}{e} \int E\, i(E)\, dE$ Where $i(E)$ is the energy distribution of the charge current.

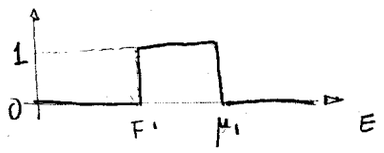
$$U = \frac{\int E\, i(E)\, dE}{\int i(E)\, dE} = \frac{e I_U}{I} \Rightarrow I_U = \frac{UI}{e}$$

since the current is conserved along x

$P_D = -\frac{I}{e} \frac{d}{dx} U$ $i(E) = \frac{2eM}{h} [f^+(E) - f^-(E)]$

for left $U = \dfrac{\int E\left[\theta(\mu_1 - E) - \theta(F' - E)\right] dE}{\int \left[\theta(\mu_1 - E) - \theta(F' - E)\right] dE} =$

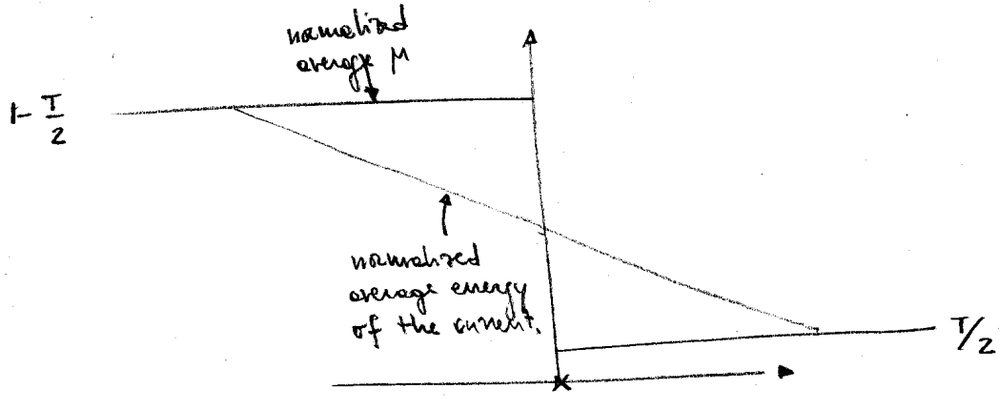
$F' = T\mu_2 + (1-T)\mu_1 \qquad \mu_2 < F' < \mu_1$

$i(E) =$ 

$$U = \frac{\mu_1^2 - F'^2}{2} \cdot \frac{1}{\mu_1 - F_1} = \frac{\mu_1 + F'}{2} = \frac{T\mu_2 + (2-T)\mu_1}{2} \overset{\text{norm}}{=} 1 - \frac{T}{2}$$

near L or R $= \dfrac{\mu_1 + \mu_2}{2}$

for Right $= \dfrac{\mu_2 + F''}{2} = \mu_2 + \dfrac{T}{2}(\mu - \mu_2) = \dfrac{T}{2}$



P_D is happening from the for left to the for right. The lenght over which this dissipation is taking place is the energy relaxation length.