Universität Regensburg | Winter Semester 09/10

Mesoscopic Physics

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Room 5.01.01
Wednesdays at 15:15

Sheet 1

1. Einstein's relation

(a) • For an electron gas in zero (quantum dot), one (quantum wire), two (2DEG), and three (bulk) dimensions, calculate the density of states $\rho(E)$ making use of the effective mass approach. Sketch the function $\rho(E)$ taking into account the level quantization for the low dimensional systems.

(b) • What changes when the electrons follow a linear dispersion relation?

(c) Derive the Einstein relation from the Drude equation for the electron conductivity in a way valid for one to three dimensions.

(d) For all dimensions, obtain an expression for the diffusion coefficient. Hint: Calculate the variance corresponding to the position of a random walk in d dimnsions. The variance is related to the diffusion coefficient by $<\Delta x^2>= D \cdot t$.

2. Liouville's theorem

• Prove the conservation of the phase space differential volume in d dimensions. Hint: verify that the determinant of the Jacobian corresponding to the infinitesimal transformation $\bar{p}' = \bar{p} + \dot{\bar{p}}\,\mathrm{d}t$ and $\bar{q}' = \bar{q} + \dot{\bar{q}}\,\mathrm{d}t$ equals 1 to first order in dt.

3. Scattering on impurities

(a) For an isotropic dispersion relation and impurity potential, and assuming elastic impurity scattering, calculate the rate of scattering $W_{kk'} = W(\epsilon_k, \hat{k} \cdot \hat{k}')$.

(b) Calculate the state and momentum relaxation times τ and τ_m, using an impurity potential of the form

$$V\left(\bar{q} - \bar{R}\right) = \begin{cases} -U & |\bar{q} - \bar{R}| < a \\ 0 & |\bar{q} - \bar{R}| \geq a \end{cases}$$

and considering the impurities randomly distributed in the sample. Hint: Remember that the difference between state and momentum relaxation times relies on the fact for the latter scattering at small angles counts less than scattering at large angles.

Frohes Schaffen!

For the exercises marked with • a written solution should be handed in by Monday of the following week at 12:00 in the post box of the course located in "*Treppenhaus 1. Etage* (*Bibliothek Physik*)".