Universität Regensburg Winter Semester 09/10

Mesoscopic Physics

Dr. Andrea Donarini Room 5.01.01
Dr. Miriam del Valle Wednesdays at 15:30

Sheet 4

1. Landau levels

• Consider a narrow conductor etched out of a wide conductor as in Fig.1 and assume a parabolic confining potential. Calculate the number of transverse modes as a function of the magnetic field, for the following cases:

(a) assuming constant Fermi energy

(b) assuming constant electron density

If the Fermi energy remains constant then the conductor can be completely depleted as the magnetic field is increased,[1] but if the electron density is assumed to remain constant then the number of modes cannot decrease to zero (at least one mode remains always occupied).[2]

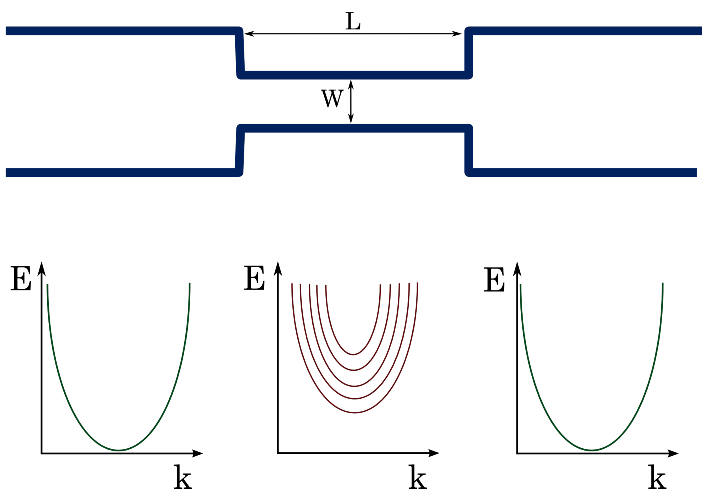

Figure 1: Junction made by the narrowing of a wide conductor. In the wide regions the transverse modes are essentially continuous, but the the narrow part they are discrete and separated in energy.

[1] B.J. van Wees *et al.*, *Phys. Rev. B* **38**, 3625 (1988)
[2] K.F. Berggren *et al.*, *Phys. Rev. B* **37**, 10118 (1988)

2. Rearrangement Theorem

The rearrangement theorem states that for the set of elements $\{g_i\}$ forming a group, if each element is multiplied from the left, or from the right, by a particular element g_j of $\{g_i\}$, then the set $\{g_i\}$ is regenerated with the elements, in general, re-ordered. Prove this theorem making use of the properties of a group, showing first that every element of the group is contained and then that it is contained only once.

3. Groups and subgroups

(a) Show that, with binary composition as multiplication, the set $\{1\ -1\ i\ -i\}$ where $i^2 = -1$, form a group G.

(b) A subset $H \subset G$, that is itself a group with the same law of binary composition, is a subgroup of G. That is, H has to satisfied closure as all other properties are automatically fulfilled. Find the subgroups of G.

4. Symmetry operations

• Consider the molecule AB_4, where the B atoms lie at the corners of a square and the A atom is at the center and is not coplanar with the B atoms.

(a) Determine the symmetry operations for this molecule.

(b) Find its multiplication table.

(c) List the subgroups.

(d) List the classes.

5. Wonderful Orthogonality Theorem

Show that every symmetry operator for every group can be represented by the (1×1) unit matrix. Is it also true that every symmetry operator for every group can be represented by the (2×2) unit matrix? If so, does such a representation satisfy the wonderful orthogonality theorem?

Frohes Schaffen!