# Mesoscopic Physics

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Room 5.01.01
Wednesdays at 15:15

## Sheet 8

### 1. Orthogonality of magnetic transverse modes

• The transverse modes of an infinite lead in presence of a magnetic field B in z-direction fulfill the Schrödinger equation

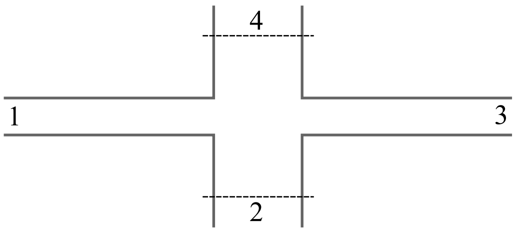
$$\left[\frac{(\hbar k + eBy)^2}{2m} - \frac{\hbar^2}{2m}\frac{d^2}{dy^2} + U(y)\right]\chi_{n,k}(y) = E_{n,k}\chi_{n,k}(y)\,. \tag{1}$$

Here $k$ is the longitudinal momentum and $U(y) = U(-y)$ is the confining potential. Prove that for $B \neq 0$,

$$\int dy\, \chi_{m,k'}(y)\left[\frac{\hbar(k+k')}{2} + eBy\right]\chi_{n,k}(y) \sim \delta_{kk'}\,. \tag{2}$$

### 2. Quantum point contacts in series

• Consider the four terminal device with two voltage probes (contacts 2 and 4):




We assume a symmetric geometry, but allow for a finite magnetic field pointing in z-direction, so that

$$\begin{aligned} G_{13} &= G_{31} \equiv G_F \\ G_{42} &= G_{24} \equiv G'_F \\ G_{21} &= G_{32} = G_{43} = G_{14} \equiv G_R \\ G_{41} &= G_{12} = G_{23} = G_{34} \equiv G_L\,. \end{aligned}$$

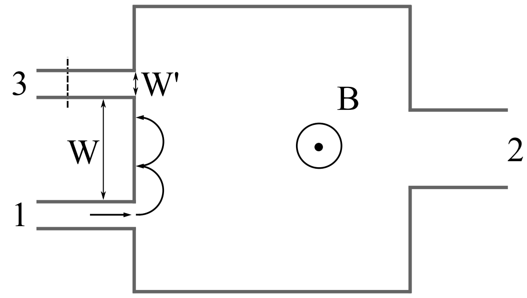
Convince yourself that the above assignments make sense, then show that the conductance is given by

$$G \equiv R^{-1}_{13,13} = \frac{1}{2}\left(\frac{2e^2}{h}M + G_F + \frac{(G_R - G_L)^2}{2G'_F + G_R + G_L}\right)\,. \tag{3}$$

$M$ is the number of propagating modes in leads 1 and 3. What happens for zero magnetic field in the coherent ($G_F = \frac{2e^2}{h}M$) and the incoherent ($G_F = 0$) limit?

## 3. Magnetic focusing

Consider the following setup with a magnetic field $B$ in z-direction:




Contact 3 is a voltage probe, contact 1 and 2 serve as source and drain.

(a) Assume a strong magnetic field so that $G_{13}$ can be neglected and show that the three terminal resistance is given by

$$R_{12,32} = \frac{G_{31}}{G_{12}G_{23}}\,. \tag{4}$$

(b) Electrons emitted from 1 follow curved trajectories as indicated. Therefore, $G_{31}$ depends strongly on $B$. In particular, $G_{13}$ is largest if the trajectory is focused into the lead at 3. At which magnetic fields can focused trajectories be expected? Give numbers for an electron density of $n = 10^{11}\,\mathrm{cm}^{-1}$ and $W = 800\,\mathrm{nm}$. What happens to $R_{12,32}$ at these fields? Sketch $R_{12,32}(B)$.

(c) At very large magnetic fields, one enters the IQH effect regime, i. e. every electron entering from 1 is absorbed into 3. Formulate a criterion for this transition.

# Frohe Weihnachten!