Quantum Theory of Condensed Matter II

Mesoscopic Physics

Prof. Ferdinand Evers
Dr. Andrea Donarini Fri 12:30, PHY 5.0.20

Sheet 5

1. Scattering time in heterostructures

In δ-doped heterostructures the charge carriers are given predominantly by the doping atoms which are situated on a plane at a distance $d \approx 100$nm from the plane of the charge carriers. The doping ions produce consequently a disordered scattering potential

$$w(q) = \left(\frac{\pi\hbar^2}{m}\right) n_\mathrm{i} e^{-2qd},$$

and $W_{\mathbf{k},\mathbf{k}'} = 2\pi\delta(\omega_\mathbf{k} - \omega_{\mathbf{k}'}) w(|\mathbf{k} - \mathbf{k}'|)$.

1. Calculate the transport relaxation time in the Born approximation.
2. Which value do you obtain for $k_\mathrm{F}\ell$, in the case $n_e = n_i$ (*i.e.* electron density equal to the impurity density) ? Assume $k_\mathrm{F} d \approx 10$. Remember that $\ell = v_\mathrm{F}\tau_\mathrm{tr}$ is the mean free path.

(4 Points)

2. Second order contribution to the self energy

Consider the selfenergy diagram:

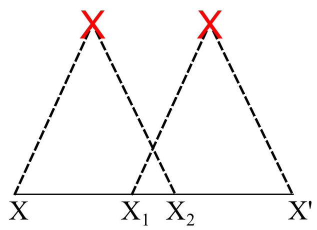

Figure 1: Example of second order impurity self energy

1. Use the diagrammatic rules from the lecture and translate the diagram into a double space integral with respect to the coordinates $\mathbf{x_1}$ and $\mathbf{x_2}$ representing the function $\Sigma_{\mathbf{x},\mathbf{x}'}$.
2. Derive an expression for the Fourier transformed $\Sigma_\mathbf{k}$. Keep in mind that, due to the translational invariance regained after impurity selfaveraging, $\Sigma_{\mathbf{x},\mathbf{x}'} = \Sigma(\mathbf{x} - \mathbf{x}')$. First express each of the potential correlators $W(\mathbf{x} - \mathbf{x}')$ in terms of their Fourier components. As a second step insert the explicit expression for the unperturbed Green's functions.
3. Finally, perform the integral over the position variables $\mathbf{x_1}, \mathbf{x_2}$. Interpret the final expression as the diagram above in momentum space. Associate to each line a momentum and show that momentum is conserved at every point where lines are intersecting.

(6 Points)

Frohes Schaffen!