Universität Regensburg Winter Term 14/15

Quantum Theory of Condensed Matter II

Mesoscopic Physics

Prof. Ferdinand Evers
Dr. Andrea Donarini Fri 12:30, PHY 5.0.20

Sheet 9

1. An electronic Fabry-Pérot interferometer

Consider a one dimensional wire with two identical scatterers located at $x = -d/2$ and $x = +d/2$, respectively. Further assume that the scattering potential is approximated by

$$U(x) = U_0[\delta(x + d/2) + \delta(x - d/2)].$$

1. Calculate the scattering and the transmission matrix for the single scatterer. Prove that the associated transmission probability reads:

$$T_\delta = \frac{\hbar^2 v^2}{\hbar^2 v^2 + U_0^2}$$

   with the electronic velocity $v = \sqrt{2E/m}$ (E is the kinetic energy of the scattered electrons).
   Hint: Recall from QM I the special matching conditions for the wavefunctions at δ potentials.

2. Calculate the transmission probability through the double δ-barrier using the combination of transfer matrices. Prove that the result reads:

$$T(E) = \frac{T_\delta^2}{1 - 2R_\delta \cos\theta + R_\delta^2}$$

   where $\theta = 2[dmv/\hbar + \arctan(\hbar v/U_0)]$ and $R_\delta = 1 - T_\delta$. Plot $T(E)$ for $U_0 = 9\,\mathrm{eV\AA}$, $d = 50\text{Å}$ and $0 \leq E \leq 250\,\mathrm{meV}$.

3. What is the maximum value for the transmission coefficient calculated in the previous point? In particular calculate the position of the resonances (*i.e.* the maxima of $T(E)$) both in the low and high energy limit, *i.e.* $\hbar v \ll U_0$ and $\hbar v \gg U_0$. Give a physical interpretation of the result.

(6 Points)

2. Strong localization

Consider the model for a disordered quantum wire, where each impurity corresponds to a δ scatterer. The goal is to show that the resistance of the wire grows exponentially with its length L:

$$\mathcal{R}(L) \approx \frac{1}{2}\left(e^{2L/L_0} - 1\right)$$

where $\mathcal{R}$ is the resistance in units of $h/(2e^2)$ and L_0 is a length scale comparable to the mean free path.

1. Start from the result obtained in the in the previous exercise and show that the (4-point) dimensionless resistance $\mathcal{R}$ for a 1D wire with two impurities can be written as:

$$\mathcal{R}_{12} \equiv \frac{R_{12}}{T_{12}} = \mathcal{R}_1 + \mathcal{R}_2 + 2\mathcal{R}_1\mathcal{R}_2 - 2\sqrt{\mathcal{R}_1\mathcal{R}_2(1+\mathcal{R}_1)(1+\mathcal{R}_2)}\cos\theta$$

   where $\mathcal{R}_i$ is the resistance R_1/T_1 of the single impurity and $\theta = 2dmv/\hbar + \arctan(\hbar v/U_1) + \arctan(\hbar v/U_2)$ and U_i the strength of the i-th impurity.
   Hint: Remember that $T_i = 1 - R_i$ and thus $T_i^{-1} = \mathcal{R}_i + 1$.

2. Argue that one can neglect the term containing the $\cos\theta$ under the assumption of i) disorder in the position of the impurities, ii) disorder in the strength of the impurity potential, iii) finite voltage and finite temperature transport. Moreover show that the Ohmic behaviour is regained in the limit of weak scatterers, *i.e.* $R_i \ll 1$.

3. Show that the arguments of the previous 2 points are applicable also to the more general case in which R_1 and R_2 are the reflection probabilities of two *groups* of scatterers.

4. Eventually, consider the situation of N groups of scatterers to which we add yet another one of resistance $\delta\mathcal{R}$. Prove the following relation:

$$\mathcal{R}(N+1) = \mathcal{R}(N) + \delta\mathcal{R} + 2\mathcal{R}(N)\delta\mathcal{R}$$

   Moreover show that in the limit in which Ohms low is valid for the single group of scatterers (*i.e.* weak impurities and sufficiently small groups) the following differential equation holds:

$$\frac{d\mathcal{R}(L)}{dL} = \frac{1}{L_0}(1 + 2\mathcal{R}(L))$$

   where $L_0 \approx \ell$, the mean free path. Prove that the solution of the equation above gives the desired length dependance of the resistance. Is Ohms law contained in the result? For which resistances (give a value is Ωs) do we expect the strong localization to take place?

(8 Points)

Frohes Schaffen!