Quantum theory of condensed matter II

Mesoscopic physics

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Tue	8:00 - 10:00	9.2.01
Fri	10:00 - 12:00	H33
Fri	12:00 - 14:00	5.0.20

Sheet 4

1. Mesoscopic beam splitter

Consider the device with three terminals sketched below, restrict yourself to a single mode per terminal and assume that time reversal symmetry holds.

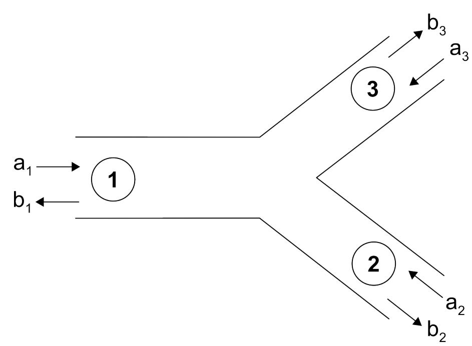

Figure 1: A mesoscopic beam splitter.

1. Assume furthermore that the device is invariant under the exchange of terminal 2 and terminal 3 and show that under these conditions the scattering matrix connecting the incoming amplitudes a_α to the outgoing ones b_α (with $\alpha = 1, 2, 3$) can be parametrized as

$$S = \begin{pmatrix} r & t & t \\ t & r' & t' \\ t & t' & r' \end{pmatrix} \tag{1}$$

where r, r', t and $t' \in \mathbb{C}$. **(2 Points)**

2. The 4 parameters of the scattering matrix S obtained in the previous point are not independent. Why? Assume all the parameters of S to be real and show that, for nonzero t, either

$$t^2 = \frac{1-r^2}{2}, \qquad r' = -\frac{1+r}{2}, \qquad t' = \frac{1-r}{2}$$

or

$$t^2 = \frac{1-r^2}{2}, \qquad r' = \frac{1-r}{2}, \qquad t' = -\frac{1+r}{2}$$

has to hold. What is the maximum value allowed for t^2? Which scattering matrices can fulfill instead the condition $t = 0$? Give a physical interpretation of the results.

(3 Points)

3. Consider now a fully symmetric system, invariant under any permutation of the three terminals. What changes in the scattering matrix S? Can r' be zero in this case?

(2 Points)

4. Assume that the terminal 1 is connected to a grounded electronic reservoir while and lead 2 and 3 are connected to a common reservoir kept at potentials V_{bias}. Using the Landauer-Büttiker formulism, express the current at terminal 1 the system as a function of the bias V_{bias} and the parameters r, r', t and t'.

(2 Points)

5. Assume like in the previous point that the reservoir 1 is grounded, while the reservoir 3 is kept to a potential V, but the potential at 2 is floating. Calculate, in terms of the transmission coefficients $T_{\alpha\beta}$ with $\alpha, \beta = 1, 2, 3$ the current flowing through terminal 1 and the floating potential of the reservoir 2. The current in terminal 2 is zero. Why?

(3 Points)

2. An electronic Fabry-Pérot interferometer

Consider a one dimensional wire with two identical scatterers located at $x = -d/2$ and $x = +d/2$, respectively. Further assume that the scattering potential is approximated by

$$U(x) = U_0[\delta(x + d/2) + \delta(x - d/2)].$$

1. Calculate the scattering matrix for the single scatterer. Prove that the associated transmission probability reads:

$$T_\delta = \frac{\hbar^2 v^2}{\hbar^2 v^2 + U_0^2}$$

with the electronic velocity $v = \sqrt{2E/m}$ (E is the kinetic energy of the scattered electrons).
Hint: Recall from QM I the special matching conditions for the wavefunctions at δ potentials.

(2 Points)

2. Calculate the transmission probability through the double δ-barrier using the combination of scattering matrices. Prove that the result reads:

$$T(E) = \frac{T_\delta^2}{1 - 2R_\delta \cos\theta + R_\delta^2}$$

where $\theta = 2[dmv/\hbar + \arctan(\hbar v/U_0)]$ and $R_\delta = 1 - T_\delta$. Plot $T(E)$ for $U_0 = 9\,\text{eV}\text{Å}$, $d = 50\text{Å}$ and $0 \leq E \leq 250\,\text{meV}$.

Hint: Do not forget to consider the free propagation of the electronic wavefunction between the scattering barriers.

(2 Points)

3. What is the maximum value for the transmission coefficient calculated in the previous point? In particular calculate the position of the resonances (*i.e.* the maxima of $T(E)$) both in the low and high energy limit, *i.e.* $\hbar v \ll U_0$ and $\hbar v \gg U_0$. Give a physical interpretation of the result.

(2 Points)

Frohes Schaffen!