University of Regensburg | Winter Term 16/17

Quantum theory of condensed matter II

Mesoscopic physics

Prof. Milena Grifoni	Tue	8:00 - 10:00	9.2.01
	Fri	10:00 - 12:00	H33
PD Dr. Andrea Donarini	Fri	12:00 - 14:00	5.0.20

Sheet 7

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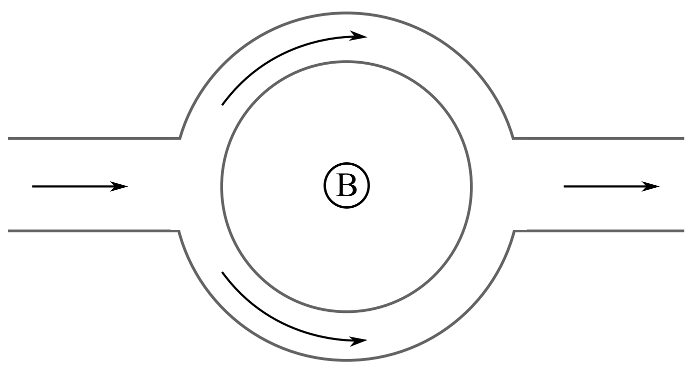
1. Mesoscopic Aharonov-Bohm effect

The conductance through a loop which is pierced by a magnetic field B oscillates as a function of the field, a phenomenon called Aharonov-Bohm effect. The essence of it can be obtained within basic scattering theory. Consider two identical beam splitters described by the scattering matrix (see exercise 1 of Sheet 4)

$$S = \begin{pmatrix} r & t & t \\ t & r' & t' \\ t & t' & r' \end{pmatrix},$$

connected in series through a ring threaded by a magnetic field as sketched in the figure below. Assume that the magnetic field is nonzero only in the middle region of the ring and thus that the electrons do not feel a Lorentz force.



1. Start with $B = 0$. Suppose that electrons acquire a phase φ when traversing either the upper or the lower branch of the ring (that is to say the "transmission" amplitude of one branch is $e^{i\varphi}$ just as it is e^{ikL} for free propagation of a distance L). Show that the transmission amplitude across the entire structure $\tilde{t}$ is given by

$$\tilde{t}(\varphi) = 2\,t^2 e^{i\varphi} \frac{1 - (r' - t')^2 e^{2i\varphi}}{1 - 2\,(r'^2 + t'^2)\,e^{2i\varphi} + (r'^2 - t'^2)^2\,e^{4i\varphi}}\,.$$

   You can use Maple or Mathematica for the algebra. Show that if r, t, r' and $t' \in \mathbb{R}$, the transmission probability reads

$$T \equiv |\tilde{t}|^2 = \frac{(1 - r^2)^2}{1 - 2\,r^2 \cos(2\varphi) + r^4}\,.$$

   Hint: Use the relations among the parameters r, t, r' and t' already derived in the exercise 1 of Sheet 4. **(2 Points)**

2. Where are the conductance resonances as a function of φ? Give a physical interpretation of the resonance condition. **(2 Points)**

3. If the magnetic field is finite, an electron moving clockwise through one of the arms acquires an additional phase ϕ, while an electron moving counterclockwise acquires an additional phase $-\phi$ with $2\,\phi = \oint \boldsymbol{A} \cdot d\boldsymbol{l} = 2\pi\Phi/\Phi_0$. Here Φ is the magnetic flux through the ring and $\Phi_0 = h/e$ is the magnetic flux quantum. Show that in this case one gets

$$\tilde{t}(\varphi, \phi) = 2\,t^2 \cos(\phi) e^{i\varphi} \frac{1 - (r' - t')^2 e^{2i\varphi}}{1 - 2\,(r'^2 + t'^2 \cos[2\phi])\, e^{2i\varphi} + (r'^2 - t'^2)^2\, e^{4i\varphi}}\,.$$

The transmission probability $|\tilde{t}(\varphi, \phi)|^2$ is an oscillating function of Φ. The fundamental frequency is given by Φ_0. These oscillations are called Aharonov-Bohm (AB) oscillations. However also higher harmonics with periods that are integer fractions of Φ_0 are present, for example the Altshuler-Aronov-Spivak (AAS) oscillations with period $\Phi_0/2$. What is the physical origin of the AB and the AAS oscillations? Plot $T(\varphi, \phi)$ for different parameters. **(2 Points)**

4. Consider the limit of a nearly closed ring $r = 1 - \Delta$ and $t' = \Delta/2$, $\Delta \ll 1$. Show that only the fundamental oscillation survives in leading order in Δ. Explain this observation. **(2 Points)**

The solution to this Sheet should be handed in by the 14$^{\text{th}}$ of December 2016.

Frohes Schaffen!