University of Regensburg Winter Term 16/17

Quantum theory of condensed matter II

Mesoscopic physics

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	Fri	10:00 - 12:00	H33
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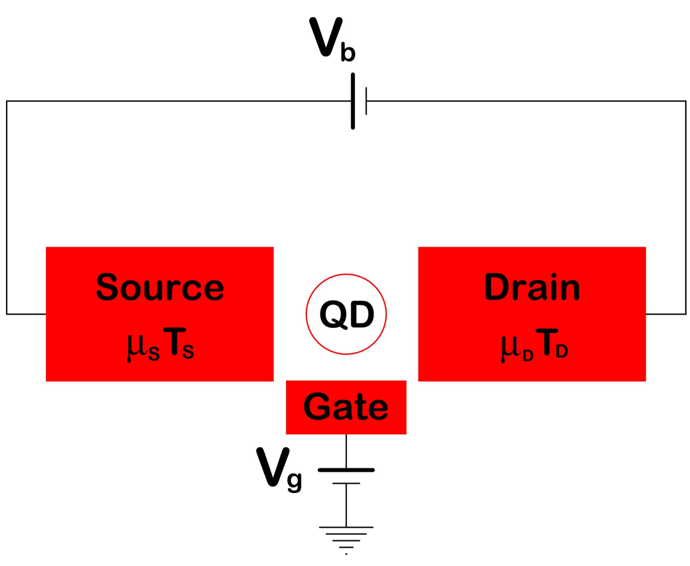
Sheet 12

1. Single electron transistor (SET)

The figure below is a schematic representation of a single electron transistor: A quantum dot is coupled to a source, a drain and a gate electrode. We describe the system via the Anderson impurity model tunnel coupled to two leads as discussed in the lecture. The chemical potentials difference of the leads can be tuned by an applied bias potential, i.e. $eV_\mathrm{b} = \mu_\mathrm{L} - \mu_\mathrm{R}$, where $e > 0$ is the modulus of the electron charge and $\mu_\mathrm{L}(\mu_\mathrm{R})$ is the left(right)-lead chemical potential. The potential drop across the structure depends on the capacitive coupling between the dot and the leads. Moreover, we introduce the effect of the gate via a modification of the Anderson Hamiltonian

$$\hat{\mathrm{H}}_\mathrm{S} = \sum_\sigma \left(\varepsilon_\mathrm{d} - e\alpha_\mathrm{G} V_\mathrm{G}\right) \hat{\mathrm{d}}^\dagger_\sigma \hat{\mathrm{d}}_\sigma + U \hat{\mathrm{N}}_\uparrow \hat{\mathrm{N}}_\downarrow,$$

where V_G is the electrostatic potential of the gate electrode, α_G is the gate coupling constant and $\hat{\mathrm{N}}_\sigma$ is the dot number operator per spin species. Assume moreover that the bias voltage drops symmetrically with respect to the dot, i.e. $\mu_\mathrm{L} = \mu_0 + eV_\mathrm{b}/2$ and $\mu_\mathrm{R} = \mu_0 - eV_\mathrm{b}/2$.



1. Following the procedure we adopted in the previous Sheets, obtain the reduced density matrix master equation

in the following form:

$$\dot{P}_0 = - \sum_{\alpha=S,D} \gamma_\alpha \left[2f_\alpha^+(\varepsilon_\mathrm{d})P_0 - \sum_\sigma f_\alpha^-(\varepsilon_\mathrm{d})P_{1\sigma} \right],$$

$$\dot{P}_{1\sigma} = - \sum_{\alpha=S,D} \gamma_\alpha \left[\left(f_\alpha^+(\varepsilon_\mathrm{d}+U) + f_\alpha^-(\varepsilon_\mathrm{d}) \right) P_{1\sigma} \right] + \\ + \sum_{\alpha=S,D} \gamma_\alpha \left[f_\alpha^+(\varepsilon_\mathrm{d}) P_0 + f_\alpha^-(\varepsilon_\mathrm{d}+U) P_2 \right],$$

$$\dot{P}_2 = - \sum_{\alpha=S,D} \gamma_\alpha \left[2f_\alpha^-(\varepsilon_\mathrm{d}+U) P_2 - \sum_\sigma f_\alpha^+(\varepsilon_\mathrm{d}+U) P_{1\sigma} \right],$$

where $f_\alpha^+(\varepsilon) \equiv [1 + e^{\beta_\alpha(\epsilon-\mu_\alpha)}]^{-1}$ and $f_\alpha^-(\varepsilon) = 1 - f_\alpha^+(\varepsilon)$.

2. Verify that the conditions for allowed tunnelling derived in the exercise 1 of Sheet 3 are the same where the Fermi functions show an inflection point. Give a physical interpretation of the result.

3. Prove that the current flowing from the α-th bath towards the impurity is given by

$$I_\alpha = \gamma_\alpha \sum_\sigma \left\{ f_\alpha^+(\varepsilon_\mathrm{d})P_0 + \left[f_\alpha^+(\varepsilon_\mathrm{d}+U) - f_\alpha^-(\varepsilon_\mathrm{d}) \right] P_{1\sigma} - f_\alpha^-(\varepsilon_\mathrm{d}+U)P_2 \right\}$$

   Hint: Consider the definition of the current as the average particle variation on the impurity.

4. Prove that, according to the previous formula, the stationary currents I_α vanish if the two baths have the same chemical potential and the same temperature.

5. Prove that, in the stationary limit, $I_S = -I_D$.

6. Sweeping the gate voltage one can change the electron number one by one. Determine the gate values at which the number of electrons in the dot changes. In such "resonant" conditions calculate the conductance.

   Hint: Define the current as $I = (I_\mathrm{S} - I_\mathrm{D})/2$ and use the current conservation condition ($I_\mathrm{D} = -I_\mathrm{S}$) we obtained in the previous point.

7. Considering the second order approximation for the current and the populations of the SET, evaluate numerically its stationary transport characteristics. Plot the populations, current and conductance ($G(V_\mathrm{b}, V_\mathrm{G}) \equiv \mathrm{d}I(V_\mathrm{b}, V_\mathrm{G})/\mathrm{d}V_\mathrm{b}$) of such a system as a function of the bias and gate voltages. Comment the result.

Frohes Schaffen!