Density Matrix Theory

Lectures	Wed	11:15 - 12:45	PHY 2.0.31
	Thu	10:00 - 11:30	PHY 2.1.29
Exercises	Fri	14:00 - 15:45	PHY 9.1.10

Sheet 7

1. Time evolution for a Markovian master equation

In this exercise, we consider the Markov master equation (1) from the last exercise sheet:

$$\begin{aligned}\dot{P}_0 &= -2\gamma f^+(\varepsilon_d)P_0 + \gamma\sum_\sigma f^-(\varepsilon_d)P_{1\sigma}\\ \dot{P}_{1\sigma} &= -\gamma[f^+(\varepsilon_d+U)+f^-(\varepsilon_d)]P_{1\sigma}\\ &\quad+\gamma f^+(\varepsilon_d)P_0+\gamma f^-(\varepsilon_d+U)P_2\\ \dot{P}_2 &= -2\gamma f^-(\varepsilon_d+U)P_2+\gamma\sum_\sigma f^+(\varepsilon_d+U)P_{1\sigma}\end{aligned} \tag{1}$$

Now we want to calculate numerically the time evolution for the populations of the many-body states of the impurity.

1. Show that the equations (1) can be cast into a matrix form $\dot{P}(t) = LP(t)$ where $P \equiv (P_0, P_{1\uparrow}, P_{1\downarrow}, P_2)^T$ and

$$L = \gamma\begin{pmatrix} -2f^+(\varepsilon_d) & f^-(\varepsilon_d) & f^-(\varepsilon_d) & 0\\ f^+(\varepsilon_d) & -f^-(\varepsilon_d)-f^+(\varepsilon_d+U) & 0 & f^-(\varepsilon_d+U)\\ f^+(\varepsilon_d) & 0 & -f^-(\varepsilon_d)-f^+(\varepsilon_d+U) & f^-(\varepsilon_d+U)\\ 0 & f^+(\varepsilon_d+U) & f^+(\varepsilon_d+U) & -2f^-(\varepsilon_d+U)\end{pmatrix}.$$

   Prove that the solution of the equation can be written in the form $P(t) = e^{Lt}P(t=0)$. Taking advantage of this algebraic formulation, calculate the numerical solution of (1).

2. Prove that, if the time is measured in units of $1/\gamma$ solutions with different tunneling rates coincide and verify this statement numerically.

3. Check that the stationary solution is reached by the system after a time corresponding to a few $1/\gamma$ and that it is independent of the initial condition.

4. Calculate the time evolution for the population vector P also with the help of one of the packages for ordinary differential equations available in Matlab. Compare the results with the previous method. Hint: There are different types of solvers. You can start by typing “help ode23” in the command line and read the documentation.

Frohes Schaffen!