Density Matrix Theory

Lectures	Wed	11:15 - 12:45	PHY 2.0.31
	Thu	10:00 - 11:30	PHY 2.1.29
Exercises	Fri	14:00 - 15:45	PHY 9.1.10

Sheet 9

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1. Current through the impurity

On the last exercise sheet an Anderson impurity model with multiple baths was discussed. Consider now the situation in which only 2 baths are in tunneling coupling with the impurity. If the chemical potentials of the two baths are maintained at a constant difference we obtain a net stationary current through the system.

1. Prove that the current flowing from the bath α towards the impurity is given by the formula:

$$I_\alpha = \gamma_\alpha \sum_\sigma \left\{ f_\alpha^+(\varepsilon_d)P_0 + [f_\alpha^+(\varepsilon_d+U) - f_\alpha^-(\varepsilon_d)]P_{1\sigma} - f_\alpha^-(\varepsilon_d+U)P_2 \right\}$$

   Hint: Start with the definition of the current as the average particle variation on the impurity.

2. Prove that, according to the previous formula, the stationary currents I_α vanish if the two baths have the same chemical potential and the same temperature.

3. Prove that, in the stationary limit, $I_1 = -I_2$ where 1 and 2 indicate the two baths.

2. Nakajima-Zwanzig in interaction picture

Consider a system-bath described by the Hamiltonian:

$$H = H_S + H_B + H_T$$

where $[H_S, N_S] = [H_B, N_B] = 0$ being N_S and N_B respectively the system and bath number operators. Moreover, assume a the tunnelling Hamiltonian H_T of the form:

$$H_T = t \sum_{ik\sigma} c_{k\sigma}^\dagger d_{i\sigma} + h.c.$$

where $c_{k\sigma}^\dagger$ creates a particle with spin σ and momentum k in the bath and $d_{i\sigma}$ destroys a particle with spin σ in the system orbital i. Prove that, if the total density matrix is factorized at the time $t = 0$ in which all representations coincide (i.e. $\rho(0) = \rho_S \otimes \rho_B$ with ρ_B the thermal equilibrium density operator) the following relation holds:

$$\mathcal{P}\dot{\rho}_I(t) = \int_0^t \mathrm{d}s\, \mathcal{P}\mathcal{L}_{T,I}(t)\mathcal{G}_{Q,I}(t,s)\mathcal{L}_{T,I}(s)\mathcal{P}\rho_I(s)$$

where

$$\mathcal{G}_{Q,I}(t,s) = T_\leftarrow \exp\left[\int_s^t \mathrm{d}t'\, \mathcal{Q}\mathcal{L}_{T,I}(t')\right], \mathcal{P}[\bullet] = \mathrm{Tr_B}\{\bullet\} \otimes \rho_B, \mathcal{Q} = 1 - \mathcal{P} \text{ and } \mathcal{L}_{T,I}(t)[\bullet] = -\tfrac{i}{\hbar}[H_{T,I}(t), \bullet]$$

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Frohes Schaffen!